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Proposal for Autonomous Decentralized Structure Formation Based on Local Interaction and Back-Diffusion Potential*

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SUMMARY Clustering technology is very important in ad hoc networks and sensor networks from the view point of reducing the traffic load and energy consumption. In this paper, we propose a new structure formation mechanism as a tool for clustering. It meets the key clustering requirements including the use of an autonomous decentralized algorithm and a consideration of the situation of individual nodes. The proposed mechanism follows the framework of autonomous decentralized control based on local interaction, in which the behavior of the whole system is indirectly controlled by appropriately designing the autonomous actions of the subsystems. As an application example, we demonstrate autonomous decentralized clustering for a two-dimensional lattice network model, and the characteristics and adaptability of the proposed method are shown. In particular, the clusters produced can reflect the environmental situation of each node given by the initial condition.

key words: *autonomous decentralized control, diffusion equation, clustering, network configuration, structure formation, local interaction*

1. Introduction

Mobile ad hoc networks (MANETs) and sensor networks are intended to work without a preset communication infrastructure and are attracting a great attention as communication tools that are useable in areas that have been subject to a severe disaster, in addition to their use at event sites. In order to utilize the infrastructure-free property and the flexibility of these networks, self-configuring mechanisms for the networks are essential. In particular, the ability to produce a well-ordered network configuration from a chaotic situation is a requirement. This paper focuses on a clustering mechanism to provide a self-configuring mechanisms. It is applicable to clustering for hierarchical routing.

Many routing technologies for MANETs have been proposed and discussed. The representative flat-type routing technologies in MANETs are classified as either proactive schemes [3], [4] or reactive schemes [5], [6]. Both are effective if the MANET does not have many mobile nodes. As the network size grows, however, these protocols have the problem that the size of the routing table stored at each individual node becomes large. As a result, the flooding overhead of route requests increases the network load. To counter this problem, a large number of studies have looked at hierarchical routing based on clustering [7]–[9]. In hierarchical routing, mobile nodes are divided into different virtual groups (clusters) and routing is based on intra-cluster and inter-cluster calculations. Here, cluster-head candidate selection is one of the important issues in clustering schemes, and some authors [10], [11] have proposed algorithms to select cluster-heads appropriately in accordance with network conditions. However, since their algorithms require non-local information, they are not strictly autonomous decentralized algorithms.

The common basic attribute demanded from these algorithms in addition to locality is adaptability to the network environment. Each node has its specific situation that influences the formation of clusters. So, it is preferable for the clustering mechanism to take the situations of individual nodes into consideration. In addition, the following flexibility is required. When there is no cluster-head candidate in the neighborhood, the mobile node itself stands as a candidate and becomes a cluster head. On the other hand, when there are two or more clusterhead candidates, we need a highly effective and efficient metric to implement optimal cluster-head selection. Examples of the possible node-specific conditions include node degree, cluster size, mobility speed, and battery energy [12].

In this paper, we introduce an autonomous decentralized clustering method that allows the nodes to act flexibly in a manner based only on the information each individual node is aware of, i.e. its individual situation. As a concrete way to compose such a clustering mechanism with autonomous and flexible properties, we adopt the strategy introduced by our framework for autonomous decentralized control based on local interaction [13]; it is a novel control mechanism for communication networks.

This framework is based on the relation between local interaction and the solution provided by a partial differential equation. In this framework, the desired behavior of the whole system is achieved by appropriately designing the autonomous operation of the subsystems; local action rules, which exist at the micro-level, are used to achieve the required state of the whole system at the macro-level. To apply this framework, we proposed flow control based on the diffusion equation, and have confirmed the achievement of congestion avoidance [14]–[16]. Our clustering method is based on our framework. However, unlike the previous au-

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tonomous algorithm, the clustering proposal of this paper can autonomously produce spatial structures of finite size.

This paper consists of the following sections. In Sect. 2, we present the framework of our proposed autonomous decentralized technology. Section 3 describes related work. In Sects. 4 and 5, we introduce the renormalization transformation of the diffusion phenomenon for a one-dimensional network model and propose an autonomous decentralized structure formation whose goals include a finite spatial size. In addition, to apply this method to a general network topology, we extend the one-dimensional network model to a general network in Sect. 6. We evaluate the characteristics of the proposed method in Sect. 7, while Sect. 8 gives the concluding remarks.

2. Framework for Autonomous Decentralized Control Based on Local-Interaction

This section briefly summarizes the concept and the framework of our autonomous decentralized control based on local interaction.

2.1 Local Interaction and Partial Differential Equation

Let us consider the one-dimensional diffusion phenomenon as an intuitive example of local interaction. When we let a few drops of black ink fall into a glass tube filled with water, the ink density distribution follows a normal distribution and the ink spreads throughout the whole tube by diffusion. In this process, the action within a minute region of water in the glass tube is very simple: the ink diffuses from the higher density side towards the lower density side. The rate of ink diffusion is proportional to the density gradient. Even though each segment acts autonomously and only local information is available, the ink density distribution throughout the glass tube eventually exhibits orderly behavior.

Let the density function (density distribution) of a certain *quantity* at time t and position x be $p(x,t)$. In the diffusion phenomenon, the flow $J(x,t)$ of that *quantity* at time t and position x is expressed as

$$J(x,t) = -\kappa \frac{\partial p(x,t)}{\partial x}, \tag{1}$$

where κ (> 0) is a positive constant. This equation means that the *quantity* flows at a rate that is proportional to the gradient of the density. When that *quantity* is not created and annihilated from this system, the continuous equation,

$$\frac{\partial p(x,t)}{\partial t} = -\frac{\partial J(x,t)}{\partial x}, \tag{2}$$

holds. From (1) and (2), we get the following equation as the temporal evolution of the density distribution:

$$\frac{\partial p(x,t)}{\partial t} = \kappa \frac{\partial^2 p(x,t)}{\partial x^2}. \tag{3}$$

This is the well-known diffusion equation.

In this equation, when the initial condition of the density distribution $p(x,0) = \delta(x - x_0)$ is a δ function at $x = x_0$, we get the solution

$$p(x,t) = N(x - x_0, 2\kappa t), \tag{4}$$

where

$$N(x,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}\, \mathrm{e}^{-\frac{x^2}{2\sigma^2}}. \tag{5}$$

$N(x,\sigma^2)$ denotes the density function as a normal distribution with mean = 0 and variance σ^2.

As (3) is a linear differential equation, we can obtain the general solution by superposing the normal distribution under even more complex initial conditions $p(x,0) = p(x)$. This yields the following general solution:

$$p(x,t) = \int_{-\infty}^{+\infty} N(x - y, 2\kappa t)\, p(y)\, \mathrm{d}y. \tag{6}$$

2.2 Principle of Autonomous Decentralized Control Based on Local Interaction

We can consider a system that is based on local interaction as a sort of autonomous decentralized control. The diffusion phenomenon is a typical example of this framework. In other words, while the subsystems run autonomously using only local information, the state of the whole system evolves into the intended orderly behavior indirectly. Our implementation of the autonomous decentralized control based on such local interaction can be summarized as follows:

- Let us consider the behavior that the state of the whole system should have (for example (4) or (6)).
- We then find the partial differential equation whose solution yields the desired behavior (for example (3)).
- Let us identify the local interaction that the partial differential equation describes (for example (1)). We then design the subsystem behavior to reproduce the required local interaction.
- As a result, even though the autonomous action of each subsystem is based only on the local information that is available to it, the state of the whole system exhibits the desired behavior as a solution of the differential equation.

It is generally known that autonomous decentralized systems may become dead locked if the subsystems are not designed properly. Our framework of autonomous decentralized control based on local interaction avoids this problem and gives us a hint as to how to design the local actions of subsystems in such a way that the whole system is pushed into the desired state. Our autonomous decentralized control approach may be summarized as follows: “by designing the behavior of subsystems at the micro-scale properly, we indirectly realize the desired behavior of the whole system at the macro-scale”.

3. Related Works

As an example of autonomous decentralized control based

on local interaction, we have already proposed a diffusion-type flow control (DFC) mechanism [14]–[16] that is based on diffusion phenomena and is suitable for fast recovery from congestion. In this control mechanism, the state of the whole network is controlled indirectly through the autonomous actions of each node. Each node manages its local traffic flow on the basis of only the local information directly available to it, by using predetermined rules. By applying DFC, the distribution of the total number of packets in each node in the network becomes uniform over time, and the network exhibits orderly behavior.

In contrast to DFC, in this paper we consider a new application of autonomous decentralized control based on local interaction that forms structures with finite spatial size. An example of our application includes autonomous decentralized clustering in sensor networks or ad hoc networks.

Many protocols addressing the issue of clustering in wireless sensor networks have been proposed [17]. As an autonomous decentralized structure formation, the bio-inspired approach is well known [18], [19]. This approach can create spatial structures in the sensor network by using the Turing pattern created from reaction-diffusion equations. This approach is based on a biological pattern formation model [20] in which two variables, that is the activation factor (activator) and the inhibiting factor (inhibitor), are modeled by differential equations, and a spatial pattern is formed by the effects of both the activator and the inhibitor. Since this approach needs to consider the temporal evolution of the two variables, there are many parameters to adjust; in one case, [18], there are seven parameters. In general, it is difficult to identify the appropriate values for these multiple parameters to yield the intended behavior. Parameter search technology is itself a research topic for reaction diffusion systems [21]. As we will see in Sect. 7, our proposed approach can form spatial structures that take account of network conditions by using relatively few parameters.

4. Renormalization Transformation of the Diffusion Phenomenon to Suit Autonomous Decentralized Control

Up to this point we have referred to the diffusion effect as an example of autonomous decentralized control based on local interaction. We now turn to consider autonomous decentralized control for creating spatial structures with finite size for patterns other than that typifying the diffusion effect.

4.1 The Temporal Evolution of Renormalization Transformation of the Diffusion Phenomenon

When we calculate the temporal evolution of density distribution $p(x,t)$, which is the solution of (3), while simultaneously shrinking (scaling down) the x axis simultaneously by the ratio of $\sqrt{2\kappa t}/\sigma$, we get the density distribution $q(x,t)$ as follows (Fig. 1):

$$q(x,t) := \frac{\sqrt{2\kappa t}}{\sigma}\, p\left(\frac{\sqrt{2\kappa t}}{\sigma}x, t\right), \tag{7}$$

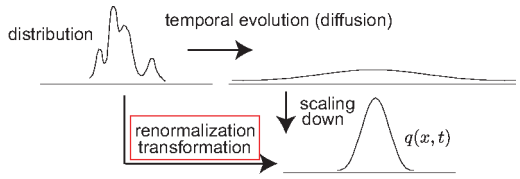

Fig. 1 Renormalization transformation of diffusion phenomenon.

where σ is a positive constant. Carrying out both the temporal evolution (diffusion) and the scaling at the same time can be regarded as a kind of renormalization transformation of the density distribution $p(x,t)$. The temporal evolution equation of the new density distribution $q(x,t)$ is given by

$$\frac{\partial}{\partial t}q(x,t) = \frac{1}{2t}\left(\frac{\partial}{\partial x}x + \sigma^2\frac{\partial^2}{\partial x^2}\right)q(x,t). \tag{8}$$

The solution of this differential equation is the following limiting distribution:

$$\lim_{t\to\infty} q(x,t) = N(x,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}\,\mathrm{e}^{-\frac{x^2}{2\sigma^2}}. \tag{9}$$

This limiting distribution comes from the fact that the solution of the diffusion equation (3) approaches a normal distribution asymptotically with time regardless of the initial conditions.

By use of this mechanism, we expect to realize a new autonomous decentralized control that can produce structures with finite spatial size. However, using (8) instead of (3) raises two problems with regard to autonomous decentralized control.

- The existence of $1/(2t)$ on the right hand side of (8) causes a slowing down of the change in the distribution over time.
- The fact that the drift term of (8) explicitly depends on x makes it necessary to set a coordinate system in the network.

We consider the latter problem in a following section.

Let us now discuss the transformation of the timescale. The distribution $q(x,t)$ in (8) should have a fast temporal evolution so that $1/(2t)$ on the right hand side of (8) can be eliminated.

If we use a constant, $c > 0$, instead of $1/(2t)$ in (8), the temporal evolution is expressed as

$$\frac{\partial}{\partial t}q(x,t) = c\left(\frac{\partial}{\partial x}x + \sigma^2\frac{\partial^2}{\partial x^2}\right)q(x,t). \tag{10}$$

The corresponding solution $q(x,t)$ is given by redefining the renormalization transformation as,

$$q(x,t) := \frac{\sqrt{2\kappa\mathrm{e}^{2ct}}}{\sigma}\, p\left(\frac{\sqrt{2\kappa\mathrm{e}^{2ct}}}{\sigma}x, \mathrm{e}^{2ct}\right). \tag{11}$$

Here, the limiting distribution (9) is invariant under the redefinition of the renormalization transformation, because $\mathrm{e}^{2ct} \to \infty$ when $t \to \infty$.

It is necessary to specify the flow corresponding to $J(x,t)$ of the continuous equation,

$$\frac{\partial}{\partial t}\,q(x,t) = -\frac{\partial}{\partial x}\,J(x,t),$$

if we are to associate (10) with autonomous decentralized control. $J(x,t)$, which satisfies this equation, is expressed as

$$J(x,t) = -c\,x\,q(x,t) - c\,\sigma^2\,\frac{\partial}{\partial x}\,q(x,t), \tag{12}$$

where c denotes the rate of temporal evolution of the density distribution and σ^2 denotes the variance of the normal distribution that is converged on. By the way, the temporal evolution Eq. (10) obtained by the renormalization transformation (11) describes the Ornstein-Uhlenbeck process [22]. For the reason given above, if we make the local action of each node follow (12), the limiting distribution is given by (9), regardless of the configuration of the initial distribution. (9) is a normal distribution that is centered on $x = 0$ and has variance σ^2, and so it is necessary to decide a special point (the origin of the coordinate system where $x = 0$) in the network and the coordinate system. The next section describes a method for designing autonomous decentralized control schemes that do not depend on any coordinate system.

5. Design of Local Interaction Based on Back Diffusion-Based Potential

As shown in the previous section, the local action of each node can be chosen as (12) for a one dimensional network. However, it requires the introduction of a coordinate system in the network and specification of the origin of the coordinate system. Since we cannot introduce a globally consistent coordinate system into general networks, a coordinate-free mechanism is required.

Creating a fundamental framework that can create spatial structures with finite size relies on balancing the effect of diffusion and drift. Accordingly, we try to keep this framework and introduce the effect of drift defined in a coordinate-free manner.

In introducing the new coordinate-free drift, we can only use the information of $q(x,t)$ itself. If we want to use other information, we should introduce another quantity in this framework. Therefore, we try to identify the coordinate-free drift defined through $q(x,t)$.

Our approach replaces x, which appears in the drift term in (12), by some function $f(x,t)$, as follows:

$$J(x,t) = -c\,f(x,t)\,q(x,t) - c\,\sigma^2\,\frac{\partial}{\partial x}\,q(x,t). \tag{13}$$

Note that this replacement does not make the limiting distribution become a normal distribution (9). Our aim is not to make a normal distribution but to form a structure with a certain finite spatial size.

The temporal evolution of distribution $q(x,t)$ that corresponds to this replacement is given by

$$\frac{\partial}{\partial t}\,q(x,t) = c\left(\frac{\partial}{\partial x}\,f(x,t) + \sigma^2\,\frac{\partial^2}{\partial x^2}\right)q(x,t). \tag{14}$$

The introduction of $f(x,t)$ eliminates the need to set a coordinate system in the network. As a more intuitive explanation, we consider the potential function $\Phi(x,t)$ instead of the function $f(x,t)$:

$$f(x,t) = -\frac{\partial \Phi(x,t)}{\partial x}. \tag{15}$$

Choosing $\Phi(x,t)$ appropriately yields autonomous decentralized control that does not depend on a coordinate system.

In general, in order to determine the potential function $\Phi(x,0)$ that will satisfy a certain global condition, we must know the macro-scale information related to the entire system, not the micro-scale information related to the system elements. This is a problem for autonomous decentralized systems that use only local information. To resolve this problem, we consider how to determine the drift term from the distribution $q(x,t)$, which is local information. Because the potential function $\Phi(x,t)$ should result in maintaining the distribution within a certain finite spatial size, contrary to the effect of diffusion, $\Phi(x,t)$ is, after discrete time Δt, given by

$$\Phi(x,t+\Delta t) = -\left(q(x,t) - \kappa'\,\frac{\partial^2 q(x,t)}{\partial x^2}\,\Delta t\right), \tag{16}$$

where $\kappa' > 0$ and $\Phi(x,t)$ is periodically renewed at Δt interval. The meaning of this equation can be expressed as follows:

- We let the time progression of the diffusion phenomenon with diffusion coefficient κ' be reversed (back diffusion).
- Next, we reverse the distribution (up and down), and regard the completed distribution as the potential.

The temporal evolution of the distribution becomes a flat state in the forward direction of time for general diffusion (Fig. 2). For back diffusion, on the other hand, the temporal evolution of the distribution goes against the direction of time and the shape of the distribution becomes sharper with time. The phenomenon that is inherent in back diffusion, a decrease in entropy, does not occur in the natural world, but it can be realized easily in the framework of autonomy decentralized control. For (16), the method of generating the potential $\Phi(x,t+\Delta t)$ by using distribution $q(x,t)$ is shown in Fig. 3.

Due to the effect of the drift term, including the potential, the peak of distribution $q(x,t)$ is emphasized and the distribution shape is sharpened. The effect of the diffusion

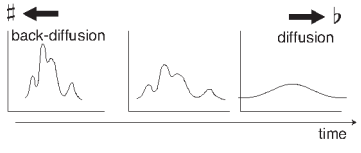

Fig. 2 Diffusion and back diffusion.

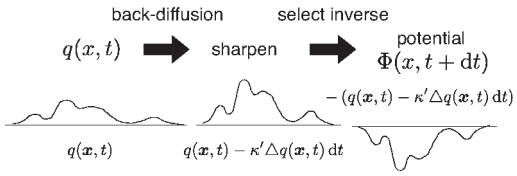

Fig. 3 Deciding potential $\Phi(x,t)$ according to back diffusion.

term, on the other hand, is to flatten the distribution. A structure with finite spatial size can be formed by balancing one effect against the other.

6. Extension to Arbitrary Network Topologies

To apply the method described in the previous section to an arbitrary network, we take the following steps:

- We first extend the one-dimensional model to n-dimensional space in preparation for applying it to the network.
- We next formulate the rule of the autonomous decentralized operation that is independent of any coordinate system, and discretize the operation in accordance with the structure of networks.

We start with space-coordinates and the differential operator of n dimensions, and replace them as follows:

$$
\begin{aligned}
&x \to \boldsymbol{x} := (x_1,\, x_2,\, \ldots,\, x_n),\\
&\frac{\partial}{\partial x} \to \boldsymbol{\nabla} := \left(\frac{\partial}{\partial x_1},\, \frac{\partial}{\partial x_2},\, \ldots,\, \frac{\partial}{\partial x_n}\right),\\
&f(x,t) \to \boldsymbol{f}(\boldsymbol{x},t) := (f_1(\boldsymbol{x},t),\, f_2(\boldsymbol{x},t),\, \ldots,\, f_n(\boldsymbol{x},t)),\\
&J(x,t) \to \boldsymbol{J}(\boldsymbol{x},t) := (J_1(\boldsymbol{x},t),\, J_2(\boldsymbol{x},t),\, \ldots,\, J_n(\boldsymbol{x},t)).
\end{aligned}
$$

This allows us to formally rewrite (13)–(16) as follows:

$$\boldsymbol{J}(\boldsymbol{x},t) = -c\,\boldsymbol{f}(\boldsymbol{x},t)\,q(\boldsymbol{x},t) - c\,\sigma^2\,\boldsymbol{\nabla}\,q(\boldsymbol{x},t), \tag{17}$$

$$\frac{\partial}{\partial t}\,q(\boldsymbol{x},t) = c\,\boldsymbol{\nabla}\cdot(\boldsymbol{f}(\boldsymbol{x},t)\,q(\boldsymbol{x},t)) + c\,\sigma^2\triangle q(\boldsymbol{x},t), \tag{18}$$

$$\boldsymbol{f}(\boldsymbol{x},t) = -\boldsymbol{\nabla}\,\Phi(\boldsymbol{x},t), \tag{19}$$

$$\Phi(\boldsymbol{x},t+\Delta t) = -\left(q(\boldsymbol{x},t) - \kappa'\triangle q(\boldsymbol{x},t)\,\Delta t\right), \tag{20}$$

where $\triangle = \boldsymbol{\nabla}^2$.

Here, it is necessary to confirm that the action rule is independent of the coordinate system and is determined by only local information. In addition, the operation rule must be extended as the action rule in order to be applied to networks. In the following, the impact of the action rule on an arbitrary network is examined concretely.

$\triangle q$ which appears in (18) represents the diffusion of $q(\boldsymbol{x},t)$. A physical meaning of $\triangle q$ is the difference between the value of q at point $\boldsymbol{x}$ and the mean value of q in the surroundings of point $\boldsymbol{x}$; note that it does not depend on any coordinate system. Therefore, it is possible to determine $\triangle q$ locally for any arbitrary network topology.

Next, we address the diffusion coefficient $\kappa := c\sigma^2$. When the diffusion phenomenon is treated as discrete, it is necessary to restrict the coefficient κ which indicates the strength of diffusion. For instance, in the one-dimensional diffusion Eq. (3), when we discretize time and space by Δx and Δt, respectively, the following condition must be satisfied [23]:

$$\kappa\,\frac{\Delta t}{(\Delta x)^2} < \frac{1}{2}.$$

In general, when we discretize n-dimensional space to a lattice space, the diffusion coefficient κ needs to satisfy

$$\kappa\,\frac{\Delta t}{(\Delta x)^2} < \frac{1}{2n}.$$

In this case, the degree of each node is $2n$. Therefore, when the above condition is generalized to the network, the diffusion coefficient should satisfy

$$\kappa\,\frac{\Delta t}{(\Delta x)^2} < \frac{1}{2\,(\max_i d_i)}, \tag{21}$$

where $(\max_i d_i)$ is the maximum node degree in the network. Note that a node with larger degree is more strongly influenced by the diffusion effects from adjacent nodes.

Though $\boldsymbol{\nabla}$ and $\boldsymbol{f}$ are vectors and have direction, they can be defined locally even in an arbitrary network, because they can be defined for each link.

Next we consider the direction of $\triangle q$ in the back diffusion shown in (20). Figure 4 illustrates the direction of back diffusion by using contour lines to represent the distribution value in the network (see the left side of the above figure). In general diffusion, the transfer of the distribution occurs towards the direction of decreasing distribution value (refer to the upper right panel in the above figure). When this is time-reversed, it becomes the flow shown in the lower left panel.

However, if the distribution quantity moves to two or

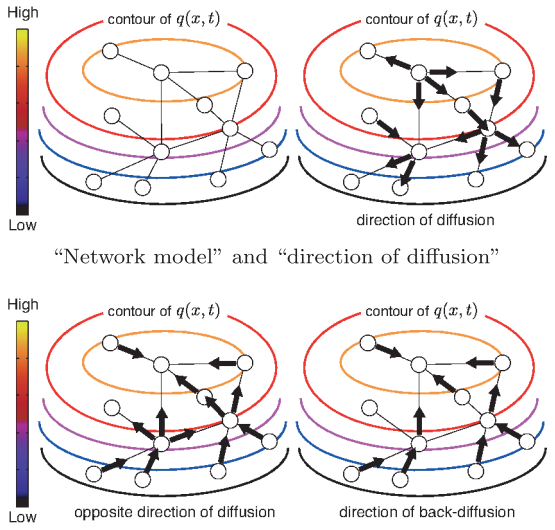

Fig. 4 Direction of back diffusion in a network.

more nodes, two or more peaks will be emphasized. In order to prevent flooding of peaks, we define back diffusion in networks as transferring the distribution towards the link of the steepest ascent direction (refer to the lower right panel in Fig. 4)[†]. Thus, back diffusion can be defined based on just the state of the node and the adjacent nodes.

From the above-mentioned consideration, the action of each node can be determined from just the relation between a node and adjacent nodes, and does not depend on a global coordinate system. One exception is the diffusion coefficient κ, which requires the maximum node degree. However, this is not a serious weakness because by assuming a sufficiently large value of the maximum node degree, the requirement (21) can be satisfied. Therefore, our approach can be applied to an arbitrary network.

Let us describe the local action rule in the network concretely. First, the set of nodes that are adjacent to node i (set of nodes that are linked to node i) is defined as N_i. In addition, we discretize time, and set the interval time of the autonomous control to be Δt.

In the following, we describe the action rule for spatial discretization that corresponds to nodes in the network and time discretization that corresponds to control timing. The distribution $q_i(t_n)$ at time t_n ($:= n \times \Delta t$) at node i changes the next time (after Δt passes) as follows:

$$q_i(t_{n+1}) = q_i(t_n) - \Delta t \sum_{j \in N_i} \left(J^{\mathrm{drift}}_{i,j}(t_n) + J^{\mathrm{diff}}_{i,j}(t_n) \right), \tag{22}$$

where $J^{\mathrm{drift}}_{i,j}(t)$ and $J^{\mathrm{diff}}_{i,j}(t)$ are the variations created by the drift effect and the diffusion effect within each unit time, respectively. $J^{\mathrm{drift}}_{i,j}(t)$ and $J^{\mathrm{diff}}_{i,j}(t)$ satisfy the following equations:

$$J^{\mathrm{drift}}_{i,j}(t_n) := \begin{cases} c\, f_{i,j}(t_n)\, q_i(t_n), & (f_{i,j}(t_n) > 0), \\ -c\, f_{j,i}(t_n)\, q_j(t_n), & (f_{j,i}(t_n) > 0), \end{cases} \tag{23}$$

$$f_{i,j}(t_n) := -(\Phi(j, t_n) - \Phi(i, t_n)), \tag{24}$$

$$J^{\mathrm{diff}}_{i,j}(t_n) := -\sigma^2 \left(q_j(t_n) - q_i(t_n) \right). \tag{25}$$

Due to the drift effect, the distribution moves in direction $i \to j$ ($j \to i$) in the case of $f_{i,j}(t_n) > 0$ ($f_{i,j}(t_n) < 0$). Here, equation $f_{i,j}(t_n) = -f_{j,i}(t_n)$ holds. The variation is proportional to the product of the velocity of the drift $f_{i,j}(t_n)$ ($f_{j,i}(t_n)$) and $q_i(t_n)$ ($q_j(t_n)$) in node i (j) (Fig. 5). The above description is formalized by (23).

The variation due to the diffusion effect is proportional to the gradient of the distribution in (25), and the direction of the diffusion is shown in Fig. 6.

Next, we explain how to determine the potential $\Phi_i(t_n)$ that is related to drift. The potential value of node i at time t_{n+1} is decided by the value of the distribution $q_i(t_n)$ and the back diffusion of $q_i(t_n)$ as follows:

$$\Phi_i(t_{n+1}) = -\left(q_i(t_n) - \kappa' \Delta t \sum_{j \in N_i} \left(J^{\mathrm{back}}_{i,j}(t_n) - J^{\mathrm{back}}_{j,i}(t_n) \right) \right), \tag{26}$$

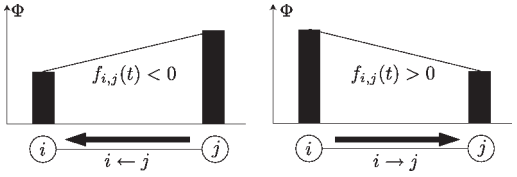

Fig. 5 Direction of the drift effect.

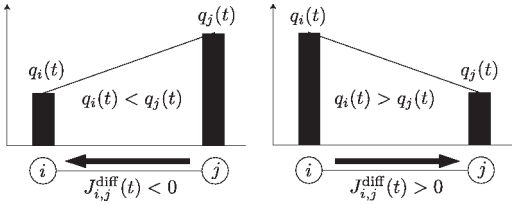

Fig. 6 Direction of the diffusion effect.

where $J^{\mathrm{back}}_{i,j}(t_n)$ is generated by the back diffusion of $q_i(t)$, and is the variation in a unit time period in the direction of node $i \to j$. The variation $J^{\mathrm{back}}_{i,j}(t_n)$ is given by

$$J^{\mathrm{back}}_{i,j}(t_n) = \begin{cases} q_j(t_n) - q_i(t_n), & (\Delta q^{\max}_i(t_n) \\ & \quad = q_j(t_n) - q_i(t_n)), \\ 0, & (\text{otherwise}). \end{cases} \tag{27}$$

$$\Delta q^{\max}_i(t_n) := \max\left(\max_{j \in N_i} \left(q_j(t_n) - q_i(t_n) \right), 0 \right). \tag{28}$$

$\Delta q^{\max}_i(t_n)$ is the difference between the distribution value of node i and the distribution value of the adjacent node j in the direction of the steepest ascent from node i. (27) explains the back diffusion shown in the lower right panel in Fig. 4. In the above-mentioned action rule for discretization in the network, local interaction is guaranteed because the summations for nodes $j \in N_i$ in the above equations involve only the nodes adjacent to node i.

To achieve the above mentioned control, it is necessary to exchange information about the values of distribution $q_j(t_n)$ for the adjacent nodes at the interval of Δt. The complexity of this information exchange does not depend on network size because the communication range is just 1 hop. Therefore, it is scalable against the number of nodes.

7. Evaluation

7.1 Evaluation of Autonomous Structure Formation Using Our Proposed Model

In this section, we apply our proposed autonomous distributed control technique to achieve clusters with a certain finite spatial size in a sensor network, and evaluate the characteristics of the proposed method. We assume a two-dimensional lattice model with 100×100 nodes to make the space structure easy to display. The network model has a torus topology to exclude the influence of the boundary

[†]Other definitions are of course possible.

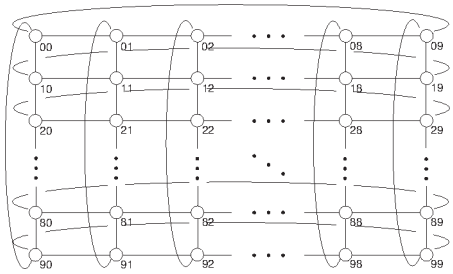

Fig. 7 Network model.

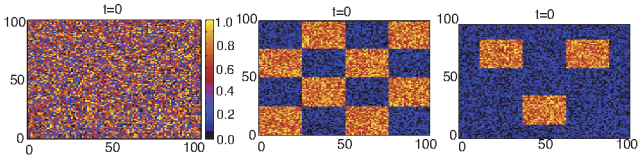

Fig. 8 Initial conditions (random, Type 1, and Type 2).

Table 1 Parameters in simulation models.

Our proposed model		Neglia model (reaction diffusion model)	
σ^2	0.8, 1.0	μ	$0.75 \times \tau$
c	0.1	ν	$0.80 \times \tau$
κ'	0.1	D_a	$0.000222273 \times \tau$
		D_h	$0.00580619 \times \tau$
		ρ_0	$0.25 \times \tau$
		ρ_1	$0.30 \times \tau$
		c	$0.50 \times \tau$

(Fig. 7). Each node has degree of 4 ($\max_i d_i = 4$) since each node has four neighbors.

We set the initial value of distribution $q_i(0)$ at random and then permit autonomous time evolution. By examining $q_i(t)$ after enough time has passed, we can confirm if the desired spatial structure can be formed from the random state that has no spatial structure.

The initial state of distribution $q_i(0)$ is shown in the left figure in Fig. 8. The initial distribution $q_i(0)$ is set to a random number uniformly distributed in [0, 1]. The color of each node shows the distribution value.

Table 1 shows the parameters used in the evaluation. Time interval Δt is set to 1. Results for cluster formation over time for two values of parameter σ^2 (0.8 and 1.0) are shown in the upper row of Fig. 9 and Fig. 10, respectively. The distributions in these panels indicate autonomous cluster structure formation. We can find a peak of a distribution by tracing the direction of the steepest ascent from a node. We consider the node as belonging to the cluster indexed by the peak node, so all nodes may thus be consistently categorized into their clusters. The proposed method forms clusters autonomously where the number and size of clusters are determined by parameter σ^2.

The panels in the lower row in Figs. 9 and 10 describe the same cluster structures by color maps. In the figures "×" denotes a cluster head, and the number of clusters corresponds to the number of cluster heads. As a result, the number of clusters in $\sigma^2 = 0.8$ and 1.0 evolves as follows:

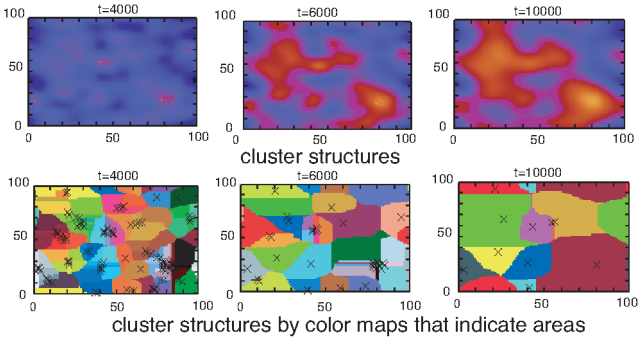

Fig. 9 Temporal variation of the cluster structure formation for the random initial distribution and cluster structures by color maps that indicate areas. ($\sigma^2 = 0.8$ in our proposed model)

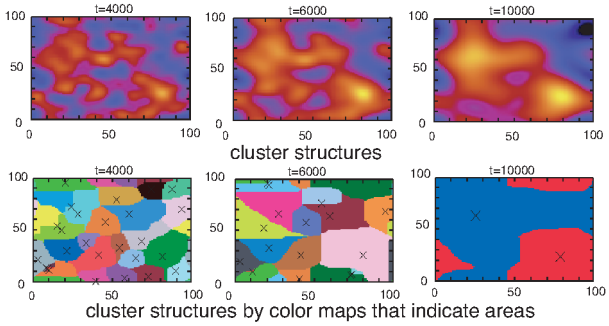

Fig. 10 Temporal variation of the cluster structure formation for the random initial distribution and cluster structures by color maps that indicate areas. ($\sigma^2 = 1.0$ in our proposed model)

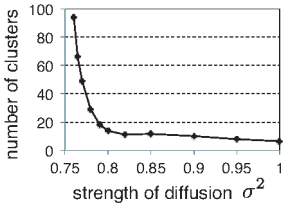

Fig. 11 Dependence of mean number of clusters on value of σ^2 in our proposed model.

- $\sigma^2 = 0.8$: 104 ($t = 4,000$), 31 ($t = 6,000$), 10 ($t = 10,000$)
- $\sigma^2 = 1.0$: 26 ($t = 4,000$), 16 ($t = 6,000$), 2 ($t = 10,000$)

The number of clusters becomes stable after $t = 10000$. It is clear that the number of clusters decreases with time, and we can see that if parameter σ^2 is small, many small clusters are created.

The mean cluster size and the cluster density are determined by balance between the strength of the diffusion effect and the strength of the drift effect. The impact of changing σ^2, which represents the strength of the diffusion effect, on the mean number of clusters is shown in Fig. 11, in which the horizontal axis is σ^2, and the vertical axis is the mean number of clusters. Figure 11 shows a result after enough

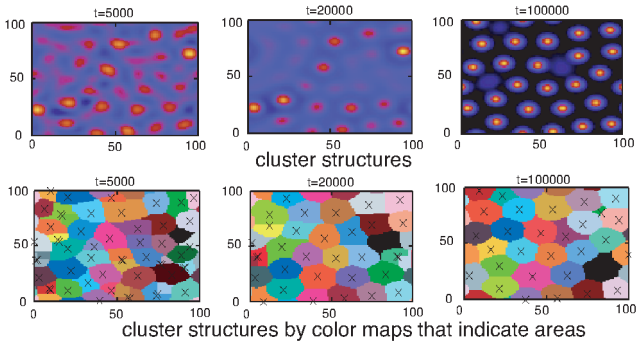

Fig. 12 Temporal variation of the cluster structure formation for the random initial distribution. (in Neglia model)

time has passed, and the number of clusters becomes stable, at least in our simulation experiments. We can see from this figure that increasing σ^2 yields fewer clusters. Thus, we are able to design for the approximate number of clusters in advance, while the cluster structure is formed autonomously regardless of the network conditions by one local action rule in each node. Hereafter, we use $\sigma^2 = 1.0$ in our proposed model.

Next, we compare our proposed model with the reaction diffusion model (Neglia model) [18]. The parameters[†] for the Neglia model are shown in Table 1. Time interval Δt is 1 as described in Sect. 7.1.

The initial value of activator $a_i(t)$ and inhibitor $h_i(t)$ are the same and the value of each node in the Neglia model is selected at random as shown in the leftmost panel in Fig. 8. Figure 12 shows the temporal evolution of the distribution $a_i(t)$ for the Neglia model. The results show that clusters are formed at almost equal spatial intervals given sufficient time. The evolution in the number of clusters is 79 ($t = 2,000$), 37 ($t = 10,000$) and 30 ($t = 100,000$).

7.2 Evaluation of Clustering Method According to Local Conditions

Next, we evaluate the structure formation for different initial conditions to match certain special situations. First of all, the two distributions shown in the middle and the right panels in Fig. 8 were taken as initial conditions; they are called Type 1 and Type 2, respectively.

The initial conditions (Type 1 and Type 2) are as follows. The areas in which the distribution is high and the areas in which the distribution is low are arranged at some spatial interval. These patterns indicate the structure of network condition with regard to the tendency to form clusters. In the areas wherein the distribution is high, $q_i(t)$ is set to a uniform random number between [0.5, 1.0], and in the areas where the distribution is low, $q_i(t)$ is set to a uniform random number between [0, 0.2]. Note that our setting of the initial condition is chosen not for making suitable cluster structure. The initial condition is given by environmental conditions e.g. residual battery power of terminals, the position of power supplies, or the node degree of mobile terminals. Therefore, the structure of clustering should reflect the characteristics of the given initial conditions.

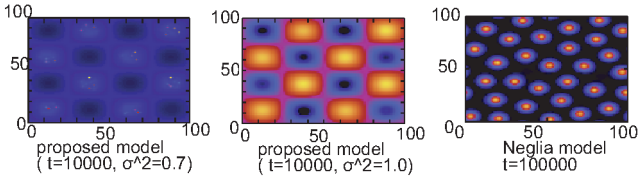

Fig. 13 Comparison of cluster formations yielded by our proposed model and Neglia model with initial condition Type 1.

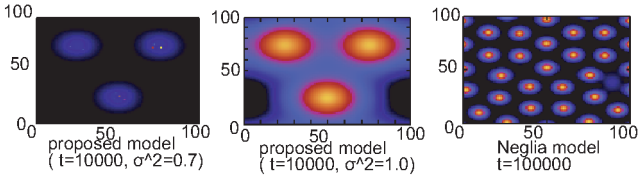

Fig. 14 Comparison of cluster formations yielded by our proposed model and Neglia model with initial condition Type 2.

The evaluation results for our proposed model and the Neglia model for initial conditions Types 1 and 2 are shown in Fig. 13 and Fig. 14, respectively. These panels show the cluster structures after sufficient time had passed.

The left panel and the panel in the centre are the results of our proposed model for $\sigma^2 = 0.7$ and 1.0, and the right panel is the result of the Neglia model. We can see from these results that the Neglia model forms a cluster structure with equal intervals, as in the previous case, even though the initial condition exhibited a strong pattern.

If we want to make clusters that divide the network into almost the same size, Neglia model may have some advantages. However, other mechanisms can be considered to make similar cluster structure. For example, we can consider that nodes with specific attributes (e.g. node ID is a multiple of 10) are cluster heads.

Our model, on the other hand, yielded a cluster structure that responded to the initial condition. In particular, it is important that different cluster structures are obtained by changing the value of parameter σ^2. The left panels of Fig. 13 and Fig. 14 contain many peaks in accordance with the fine structure of the initial condition, and the center panels of Fig. 13 and Fig. 14 contain fewer peaks in accordance with the coarse grained structure of the initial condition. Therefore, a fine cluster structure or coarse grained cluster structure can be formed by a small or larger σ^2.

If we can tune the values of parameters for Neglia model to reflect the initial condition, the Neglia model may yield cluster structures that reflect the characteristics of the initial condition. However, tuning the value of the parameter everywhere in the network demands that we know global information of the network. Such a mechanism is not an

[†]Though $\Delta t = 0.003$ ($= \tau$) in Neglia model [18], we set $\Delta t = 1$ to allow comparison with our proposed model. For such a modification of Δt for Neglia model, we revise the other parameters except Δt.

autonomous decentralized mechanism.

8. Conclusions

Autonomous decentralized control based on the principle of local interaction offers a general framework in which a local interaction rule at the micro-level yields orderly behavior of the entire system at the macro-level. Our previous study considered the diffusion effect for autonomous decentralized control based on local interaction, and focused on load-balancing. This paper has considered a new type of autonomous decentralized control that can create structures with finite spatial size. The structures obtained represent a different macro-level behavior from that obtained from the diffusion effect. First, we introduced the renormalization transformation of the diffusion phenomenon for a one-dimensional network model and considered autonomous structure formation with a certain finite spatial size. Next we formulated a rule for autonomous decentralized operation that does not depend on the use of a coordinate system. In addition, to apply this method to a general network topology, we extended the one-dimensional network model to n-dimensional Euclid space. We used the discretization method to derive a control rule that can be applied to general networks. As an application example, we introduced a technique yielding autonomous decentralized structure formation in a two-dimensional lattice network and confirmed the feasibility of the autonomous decentralized clustering mechanism for sensor networks. Future studies will investigate the characteristics of our structure formation mechanism for different values of the parameters used in our proposal and determine the appropriate parameter values for different network conditions. In addition, we will establish a dynamic clustering technology that takes the movement of nodes into consideration.

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