PAPER

Congestion Detection and CAC for ABR Services Using Allan Variance

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SUMMARY Recently, ABR has been attracting attention as a new service category of ATM, and the methodology to realize ABR is being actively discussed in the ATM Forum. ABR is expected to become a suitable class for supporting LAN services on ATM networks. To this end, a technical foundation must be established in which bandwidth is effectively utilized and quality is guaranteed. In order for ABR to use a portion of the bandwidth that is not used by high-priority classes (CBR, VBR), it is necessary to appropriately estimate the unused portion of the bandwidth. Due to the fact that the unused portion of the bandwidth in ATM networks fluctuates, such fluctuations must be taken into account. This paper describes ABR connection admission control and design of the congestion detecting point in an ABR buffer using Allan variance of the unused portion of the bandwidth.

key words: *ATM, LAN, ABR, traffic control, congestion control, Allan variance, Fokker-Plank equation*

1. Introduction

Asynchronous Transfer Mode (ATM) networks must support many kinds of traffic accompanied by a variety of services and they must guarantee quality of service (QoS). Traffic control is therefore a key technology to attain such requirements. Recently, Available Bit Rate (ABR) has been attracting attention as a new service category of ATM, and the methodology to realize ABR is being actively discussed in the ATM Forum. ABR is expected to become a suitable class for supporting LAN services (Ethernet, FDDI, etc.) on ATM networks. To this end, a technical foundation must be established in which bandwidth is effectively utilized and quality is guaranteed.

The most remarkable characteristic of ABR is that it controls the rate of cell emission. The cell emission rate of each ABR connection is regulated to a particular rate depending on the utilization of the networks. Although there are some rate control schemes proposed in the ATM Forum [1], a scheme, called the Explicit Rate (ER) mode, in which the network explicitly specifies the cell emission rate to the user using ER field of Resource Management (RM) cell is the most expected. Another scheme, called EFCI mode, controls the rate using EFCI bit of the user cell, and is also strongly supported in the ATM Forum. Regardless of which rate control scheme is used, there are general regulations for usage. The cell emission rate is generally controlled in such a way that the rate increases when the networks have light traffic and the rate decreases when the networks have heavy traffic. Although the networks are heavily congested, the networks must guarantee cell emission at the predefined Minimum Cell Rate (MCR). Therefore, no control scheme can specify a rate lower than the MCR. MCR is specified by negotiations between a user and a network at the connection setup. Whether a network is able to support MCR or not is one of the most important criteria for connection admission control of the ABR class.

When ABR services are first introduced, MCR may be set at 0. However, in the second stage of ABR services, demand for MCRs greater than 0 will increase. This is because user demands such as real-time services on data networks are currently increasing, for example, future protocols in the network layer ST-II [2] or Internet Protocol (IP) version 6 [3], should support a stream type of traffic.

Figure 1 shows two possible ways to support ABR services. In Type-1, ABR is supported using an exclusive bandwidth. Connections belonging to non-ABR classes, which have a higher-priority than ABR, use other bandwidth. These bandwidths are strictly separated. In Type-2, ABR shares a bandwidth with the high-priority classes. ABR uses the portion of the bandwidth that is not used by the high-priority classes. In this type, it is expected that bandwidth can be utilized effectively and traditional LAN services can be inexpensively supported on ATM networks. In order for ABR to use a portion of the bandwidth that is not used by high-priority classes, it is necessary to appropriately estimate the unused portion of the bandwidth. Since the unused portion of the bandwidth in ATM networks fluctuates, such fluctuations must be taken into account. Therefore, characteristics of the high-priority class traffic are closely related to the estimation of the unused portion of the bandwidth.

In addition, capacity of the ABR buffer is also related to the estimation of the unused portion of the bandwidth. This can be understood through the following example. Consider the average usage of the bandwidth caused by high-priority traffic as $\rho^{\rm h} = 0.6$. In order for ABR to use the rest of the bandwidth $1-\rho^{\rm h} = 0.4$ completely, it is necessary to prepare an ABR buffer

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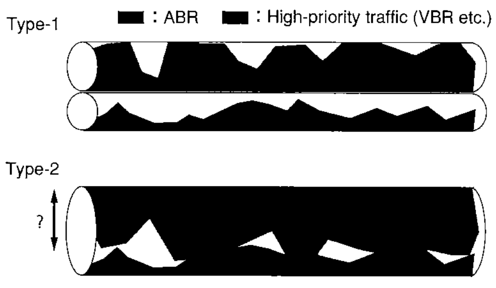

Fig. 1 Use of bandwidth for ABR.

with a large capacity.

This paper describes a method for estimating the unused portion of the bandwidth using Allan variance [4], [5]. Based on this, we propose two frameworks that determine whether the network can support MCR or not. One is using Chebyshev inequality [6] and the other is using the diffusion approximation [7]. In addition, it describes ABR Connection Admission Control (CAC) and design of the congestion detecting point in an ABR buffer using the diffusion approximation.

2. Estimating the Unused Portion of the Bandwidth

This section presents the estimation of the bandwidth that is unused by the high-priority classes and is the available bandwidth for ABR [6]. Our model is as follows: There are two classes of traffic, one is high-priority class, e.g., Variable Bit Rate (VBR) and Constant Bit Rate (CBR). The other is ABR class. Cells of ABR class at multiplexor in ATM-SW are only emitted when there are no cells of high-priority class. Consequently, ABR traffic uses the bandwidth unused by the high-priority traffic.

Since the only traffic parameters that we can control are UPC parameters, we assume that traffic characteristics of the VBR connections comply with the Generic Cell Rate Algorithm (GCRA) specified in the ATM Forum [8]. It is not applicable to take the other assumption, e.g., MMPP or Poisson process because UPC cannot decide whether the assumption is valid or not. We regard the case of a CBR connection as a special VBR connection, that is, in which the peak rate is equal to the average rate, and CBR connections also comply with the GCRA.

2.1 Generic Cell Rate Algorithm

GCRA is a flow control algorithm based on the Leaky Bucket and is described using four parameters: Peak Cell Rate (PCR), Sustainable Cell Rate (SCR), Cell Delay Variation Tolerance (CDVT), and Burst Tolerance (BT). PCR is the maximum cell-emitting rate permitted to the terminal. It is defined as the reciprocal number of the minimum interval between successive cell emission times. SCR is the maximum value of the average cell emitting rate permitted to the terminal. SCR is not greater than PCR. BT specifies the Maximum Burst Size (MBS) [8] which is denoted as

$$\mathrm{MBS} = \left[1 + \frac{\mathrm{BT}}{1/\mathrm{SCR} - 1/\mathrm{PCR}}\right],$$

where MBS means the maximum number of cells that are successively emitted at PCR, and $[x]$ is the integer part of x. CDVT is a parameter related to Cell Delay Variation (CDV). For simplicity, we do not take CDV into account and do not use CDVT in this paper.

Although CBR is not used to specify SCR and BT, we set SCR $=$ PCR, BT $= 0$ and treat it as a special VBR case.

We assume that each connection, belonging to a high-priority classes (CBR, VBR), is mutually independent. There are n high-priority connections in the transmission link (or a VP). For each connection i ($i = 1, 2, ..., n$), the network can obtain parameters, PCR P_i, SCR S_i, and BT B_i. Here, time is divided into fixed-length slots, and a single slot corresponds to the transmission time of an ATM cell. The slot length is defined as L/C, where L [bit] is the cell length and C [bps] is the capacity of the transmission link. We adopt the slot length as a unit of time.

2.2 Set of Source Traffic Patterns

This section describes sets of traffic patterns of the high-priority connections that comply with GCRA.

In order to evaluate the worst for ABR, we limit source traffic patterns to the following patterns complying with GCRA. For high-priority connection i, we consider traffic patterns which satisfy the following three conditions:

- actual maximum cell rate is PCR, P_i,
- actual average cell rate is SCR, S_i, and
- the maximum size of a bursts with PCR are included.

We name the traffic patterns as ceiling patterns, and define that a set of all ceiling patterns as $\Sigma_i = \Sigma_i(P_i, S_i, B_i)$.

The maximum size of burst with PCR P_i is

$$W_i^{\mathrm{on}} := \frac{B_i S_i + 1}{P_i - S_i}$$

slots. The interval of these bursts is

$$W_i(x_i) = \frac{B_i S_i + 1}{P_i - S_i} + \frac{B_i S_i + 1}{S_i - x_i} \qquad (1)$$

slots, where x_i denotes the average cell rate between the end of some burst and the head of the next burst. The minimum interval is

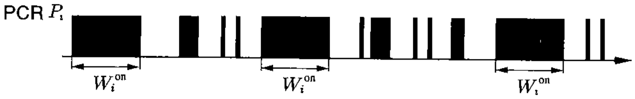

Fig. 2 Example of ceiling pattern.

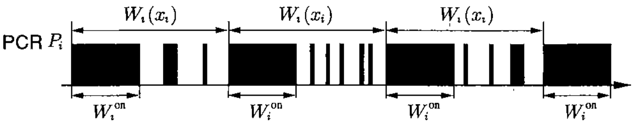

Fig. 3 Example of periodic pattern.

$$W_i(0) = \frac{B_i S_i + 1}{P_i - S_i} + \frac{B_i S_i + 1}{S_i}$$

when $x_i = 0$.

We also define subset $\Sigma_{(\mathrm{P})i}(P_i, S_i, B_i) \subset \Sigma_i$. Set of periodic patterns, $\Sigma_{(\mathrm{P})i}(P_i, S_i, B_i)$, is a set of all ceiling patterns that have constant intervals between consecutive maximum bursts. Therefore, $\varsigma \in \Sigma_{(\mathrm{P})i}$ has constant period $W_i(x_i)$ and, conversely, a parameter x_i determines the period. Examples of ceiling patterns and periodic patterns are shown in Figs. 2 and 3, respectively.

2.3 Allan Variance of the Unused Portion of the Bandwidth

2.3.1 Introducing Allan Variance of the Utilization

This section gives the index of the fluctuation of the unused portion of the bandwidth. For a ceiling pattern $\varsigma \in \Sigma_i$, we define the average utilization of the transmission link caused by high-priority connection i, and observed during successive t slots, to be $X_i(t, \varsigma)$. Set an arbitrary time to slot 0 and let $N_i(t, \varsigma)$ be the number of arriving cells during slot interval $[0, t)$. Then $X_i(t, \varsigma)$ is expressed as

$$X_i(t, \varsigma) := \frac{N_i(t, \varsigma)}{t},$$

and represents a random variable. $X_i(t, \varsigma)$ satisfies

$$E[X_i(t, \varsigma)] = S_i.$$

In order to know the fluctuation of the unused portion of the bandwidth, investigation of the behavior of $X_i(t, \varsigma)$ with respect to t is important. In this paper, we investigate the fluctuation of the unused portion of the bandwidth using Allan variance [4]. Allan variance, originally introduced as a fluctuation index for the frequency of an atomic clock, is defined as the variance of finite-time average. It is closely related to the spectrum analysis using the Fourier transformation [5]. Since Allan variance is, however, directly dependent on time, it is expected that estimating the unused portion of the bandwidth or ABR buffer design using the Allan variance is easier than using spectrum analysis. We define the Allan variance of the transmission-link utilization caused by high-priority connection i with a traffic pattern $\varsigma \in \Sigma_i$ as

$$\sigma_i^2(t, \varsigma) := E\left[(X_i(t, \varsigma) - S_i)^2\right].$$

Next, we investigate the behavior of $\sigma_i^2(t, \varsigma)$ with respect to t. $\sigma_i^2(t, \varsigma)$ has the following property.

Theorem 1: For any $\varsigma \in \Sigma_i$,

$$\sigma_i^2(t, \varsigma) = O(t^{-2}).$$

Proof: See Appendix A. □

2.3.2 Allan Variance of the Source Traffic

We only know GCRA parameters, not ς. Therefore, we define Allan variance, when GCRA parameters are given, as maximizing $\sigma_i^2(t, \varsigma)$ with respect to all ceiling patterns $\varsigma \in \Sigma_i$ as

$$\sigma_i^2(t) := \sup_{\varsigma \in \Sigma_i} \sigma_i^2(t, \varsigma). \quad (2)$$

Similarly, we define Allan variance that maximizing $\sigma_i^2(t, \varsigma)$ with respect to only periodic patterns $\varsigma \in \Sigma_{(\mathrm{P})i}$,

$$\sigma_{(\mathrm{P})i}^2(t) := \sup_{\varsigma \in \Sigma_{(\mathrm{P})i}} \sigma_i^2(t, \varsigma). \quad (3)$$

In addition, we also define the variances of the number of arriving cells during $[0, t)$ as

$$Var(N_i(t)) := t^2 \sigma_i^2(t) = t^2 \sup_{\varsigma \in \Sigma_i} \sigma_i^2(t, \varsigma),$$

and

$$Var(N_{(\mathrm{P})i}(t)) := t^2 \sigma_{(\mathrm{P})i}^2(t) = t^2 \sup_{\varsigma \in \Sigma_{(\mathrm{P})i}} \sigma_i^2(t, \varsigma).$$

According to the physical meaning of $Var(N_{(\mathrm{P})i}(t))$, it is natural to take the following assumption.

Assumption 1: $Var(N_{(\mathrm{P})i}(t))$ monotonously increases with respect to t.

Each periodic pattern $\varsigma \in \Sigma_{(\mathrm{P})i}$ has constant burst interval. Therefore, for fixed ς, x_i in Eq. (1) is constant for each period. Using a fluid model, explicit expression of $\sigma_i^2(t, \varsigma)$ for $\varsigma \in \Sigma_{(\mathrm{P})i}$, in asymptotic region $t \gg 1$, is shown in Appendix B. In addition, using the explicit form, we have the asymptotic behavior of $\sigma_{(\mathrm{P})i}^2(t)$ as the following theorem.

Theorem 2: For Allan variance of the periodic patterns $\varsigma \in \Sigma_{(\mathrm{P})i}$,

$$\sigma_{(\mathrm{P})i}^2(t) = O(t^{-2}).$$

Proof: See Appendix C. □

Although we obtained the asymptotic behavior of $\sigma_{(\mathrm{P})i}^2(t)$, our interest is not in $\sigma_{(\mathrm{P})i}^2(t)$ but in $\sigma_i^2(t)$. The following theorem guarantees that the behavior of $\sigma_{(\mathrm{P})i}^2(t)$ obtained above also applies for $\sigma_i^2(t)$.

Theorem 3:

$$\sigma_i^2(t) = \sigma_{(\mathrm{p})i}^2(t).$$

Proof: See Appendix D. □

Therefore, from theorem 2, it exists a constant $\alpha_{(\mathrm{p})i}$ such as

$$\sigma_{(\mathrm{p})i}^2(t) \leqq \alpha_{(\mathrm{p})i} t^{-2} \quad t \gg 1,$$

and if we choose

$$\alpha_i := \inf \alpha_{(\mathrm{p})i},$$

then, from theorem 3, it is the minimum value satisfying

$$\sigma_i^2(t) \leqq \alpha_i t^{-2} \quad t \gg 1.$$

2.3.3 Example of Deriving $\sigma_i^2(t)$

Using a numerical example, we show how to derive the asymptotic behavior of $\sigma_i^2(t)$ with respect to t where $t = 1$ and $t \gg 1$.

When $t = 1$, $\sigma_i^2(t)$ is easily calculated as

$$\sigma_i^2(1) = S_i(1 - S_i).$$

Note that this is independent of P_i and B_i.

Next, we investigate the case of $t \gg 1$. Consider a example of high-priority connection i, and its PCR P_i, SCR S_i, and BT B_i as 15 Mbps, 1.5 Mbps, and 10/SCR, respectively. The capacity of the transmission link is 150 Mbps. Figure 4 shows the asymptotic behavior of Allan variance $\sigma_i^2(t, \varsigma)$ with respect to periodic patterns $\varsigma \in \Sigma_{(\mathrm{p})i}$ with some values of x_i. This is derived from the explicit expression in Appendix B. The period of each ς is determined by a parameter x_i. From Fig. 4, we can recognize that $\sigma_i^2(t, \varsigma)$ decreases with vibration with respect to t. This vibration is a consequence of the periodicity in the traffic pattern.

From Eq. (3), we choose $\sigma_{(\mathrm{p})i}^2(t)$ as the maximum value, or envelope, of $\sigma_i^2(t, \varsigma)$ with respect to $\varsigma \in \Sigma_{(\mathrm{p})i}$ or x_i. Figure 5 shows the behavior of $\sigma_{(\mathrm{p})i}^2(t)$ with respect to t. From theorem 2, $\sigma_{(\mathrm{p})i}^2(t)$ decreases as $O(t^{-2})$. To visualize the theorem, Fig. 6 shows the behavior of the product of $\sigma_{(\mathrm{p})i}^2(t)$ and t^2 based on the previous numerical example. In addition, from theorem 3, $\sigma_{(\mathrm{p})i}^2(t)$ is equivalent to $\sigma_i^2(t)$. Therefore, we can have the asymptotic behavior of $\sigma_i^2(t)$ using only the periodic patterns of source traffic.

2.3.4 Allan Variance of Aggregate Traffic

This section shows the Allan variance of utilization caused by aggregated traffic.

The Allan variance of all high-priority connections, at a multiplexor in ATM-SW, is denoted as $\sigma^2(t)$. We investigate the asymptotic behavior of $\sigma^2(t)$. When $t = 1$, it is easily calculate as

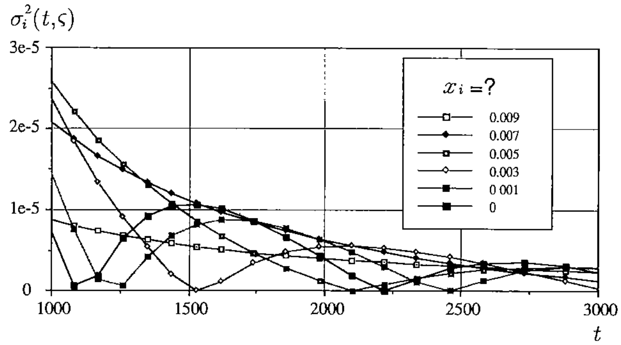

Fig. 4 Asymptotic behavior of $\sigma_i^2(t, \varsigma)$ with respect to t.

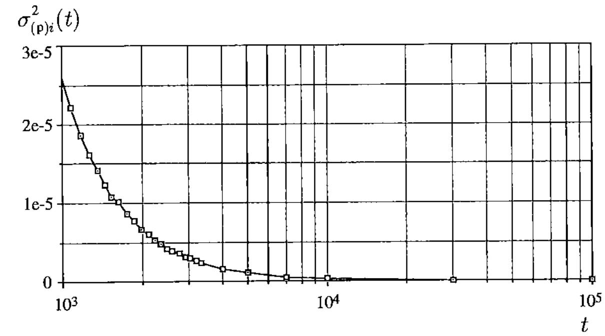

Fig. 5 Asymptotic behavior of $\sigma_{(\mathrm{p})i}^2(t)$ with respect to t.

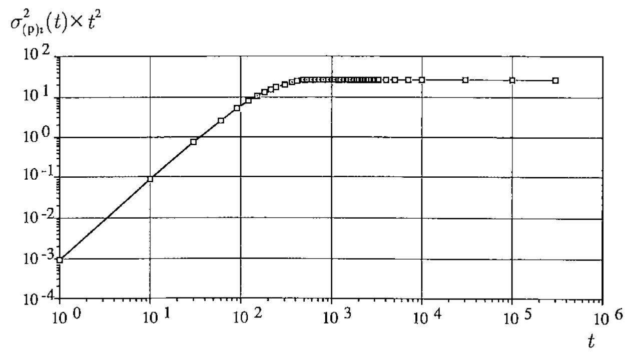

Fig. 6 Asymptotic behavior of $\sigma_{(\mathrm{p})i}^2(t) \times t^2$ with respect to t.

$$\sigma^2(1) = \rho^{\mathrm{h}}(1 - \rho^{\mathrm{h}}), \tag{4}$$

where, $\rho^{\mathrm{h}} := \sum_{i=1}^{n} S_i$ is the utilization caused by all high-priority connections.

On the other hand, when $t \gg 1$, deformation of the source traffic is negligible. This is because t is too longer than a capacity of output buffer in multiplexor. Therefore, we have

$$\sigma^2(t) = \alpha t^{-2}, \tag{5}$$

where, $\alpha := \sum_{i=1}^{n} \alpha_i$.

In addition, we adopt the following assumption;

Assumption 2: $\sigma^2(t)$ monotonously decrease with respect to t.

3. Estimating the Available Bandwidth and CAC for ABR

This section shows two frameworks for determining

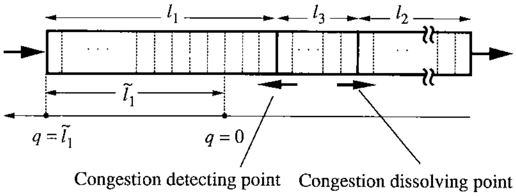

Fig. 7 ABR buffer model.

whether the available bandwidth for the ABR class is sufficient or not. One is using Chebyshev inequality and the other is the diffusion approximation using Fokker-Plank equation.

In ATM-SW, we consider a cell buffer for ABR cells that is different from the cell buffer for high-priority connections. ABR cells are emitted only at a time when no waiting cells belonging to high-priority connections are in the ATM-SW. As a result, ABR utilize the unused portion of the bandwidth.

Figure 7 shows the cell buffer for ABR. We assume the judgment criterion for limiting the ABR cell-rate to MCR is the number of cells waiting in the ABR buffer. We call this criterion the congestion detecting point. Similarly, the judgment criterion for removing this limitation is also the number of cells waiting in the ABR buffer and we call this criterion the congestion dissolving point. In this paper, congestion means to limit the ABR cell-rate to MCR.

In Fig. 7, l_1 denotes the capacity of the ABR buffer between the congestion detecting point and the fully occupied point. l_2 denotes the capacity of the ABR buffer between the congestion dissolving point and the completely vacant point. l_3 denotes the rest of the capacity for the ABR buffer.

When congestion is detected, the rate control mechanism works to dissolve congestion. In the mechanism, the cell rate of the ABR connections is limited to MCR. Control delay, the delay between the time when congestion is detected and the time when ABR connections are limited to MCR, is dependent on the rate control mechanism or network size. We define l_1^c as the capacity of the ABR buffer to prepare for the control delay. In this paper, we consider the capacity of $\tilde{l}_1 = l_1 - l_1^c$ of the ABR buffer.

In ABR, the networks must guarantee a cell emission rate at MCR despite whether the networks are congested or not. However, since ABR uses the unused portion of the bandwidth, the emission interval of ABR cells is not constant. Therefore, in order to estimate the available bandwidth for the ABR class, it is necessary to estimate the available bandwidth for ABR while still guaranteeing MCR.

3.1 Variance of Queue Length in an ABR Buffer

When we know the queue length of ABR buffer at slot $t = 0$ and all ABR connections are limited to MCR, we consider the variance, $V(T)$, of the queue length at slot $t = T$. $V(0) = 0$ when $t = 0$, and using the Allan variance of the unused portion of the bandwidth, $V(t)$ is denoted as

$$V(t) = \sigma^2(t)\, t^2.$$

Applying theorem 2 and 3 to $\sigma^2(t)$, and from Eq. (5), $V(t)$, in the asymptotic region $t \gg 1$, is denoted as

$$V(t) = \alpha \quad t \gg 1.$$

In case of $t = 1$, from Eq. (4), we have

$$V(1) = \beta,$$

where $\beta := \rho^{\mathrm{h}}(1 - \rho^{\mathrm{h}})$.

Next, we consider the intersection of $\beta\, t^2$ and α. The value of t at point is denoted as

$$T_1 = \sqrt{\frac{\alpha}{\beta}}\ .$$

We define $\widetilde{V}(t)$ such that

$$\widetilde{V}(t) = \begin{cases} \beta\, t^2 & t \in [0, T_1), \\ \alpha & t \in [T_1, \infty), \end{cases} \tag{6}$$

and approximate $V(t)$ using $\widetilde{V}(t)$. $\widetilde{V}(t)$ gives an overestimation of $V(t)$ from the following property:

Theorem 4: For any t,

$$V(t) \leq \widetilde{V}(t).$$

Proof: For $t \in [0, T_1)$, the proof is obvious from assumption 2. For $t \in [T_1, \infty)$, it is obvious from assumption 1, theorem 2 and 3. □

3.2 Estimating the Available Bandwidth and CAC

After congestion is detected, the rate control mechanism limits the cell emission rate of the ABR connection j ($j = 1, 2, \ldots, m$) to MCR M_j. We choose this time to be $t = 0$. Then the average transmission-link utilization caused by ABR traffic, ρ^{a}, is denoted as

$$\rho^{\mathrm{a}} := \sum_{j=1}^{m} M_j.$$

Consider the state in which the ABR buffer overflows and cell loss occurs. We call this heavy congestion. Since heavy congestion does not occur before $\tilde{l}_1 + 1$ ABR cells arrive, we define T_0 as

$$T_0 := \frac{\tilde{l}_1 + 1}{\rho^{\mathrm{a}}}. \tag{7}$$

T_0 means the minimum slot in which heavy congestion occurs.

3.2.1 Estimating the Available Bandwidth and CAC Using Chebyshev Inequality

This section gives a estimation of the probability of the transmission link being heavily congested at $t = T$ by applying Chebyshev inequality [9] to the queue length of the ABR buffer. The average number of cells emitted during $[0, T)$ is $(1-\rho^{\mathrm{h}})T$, and its variance is $V(T)$. Therefore, the probability of the transmission link being heavily congested at $t = T$, $P_{\mathrm{loss}}(T)$, is upper-bounded by

$$P_{\mathrm{loss}}(T) \leqq \frac{\widetilde{V}(T)}{\left\{(\widetilde{l}_1+1)+(1-\rho^{\mathrm{a}}-\rho^{\mathrm{h}})T\right\}^2}.$$

Since heavy congestion does not occur before $\widetilde{l}_1+1$ ABR cells arrive, we do not need to take time T such as

$$T < T_0 := (\widetilde{l}_1+1)/\rho^{\mathrm{a}}$$

into account. We estimate the probability of the transmission link becoming heavily congested, $\mathbf{P}_{\mathrm{loss}}$, as

$$\mathbf{P}_{\mathrm{loss}} = \max_{T \geqq T_0} P_{\mathrm{loss}}(T). \tag{8}$$

Using $\widetilde{T} := \max(T_0, T_1)$, "max" in Eq. (8) can be removed and we get

$$\mathbf{P}_{\mathrm{loss}} = P_{\mathrm{loss}}(\widetilde{T}). \tag{9}$$

The procedure of ABR CAC using Eq. (9) is as follows:

1. When ABR connection j setup is requested, the network obtains MCR M_j and calculates $\mathbf{P}_{\mathrm{loss}}$ using Eq. (9).
2. If $\mathbf{P}_{\mathrm{loss}}$ is greater than the predefined value, the network rejects the connection. Otherwise, the connection setup request is accepted.

3.2.2 Estimating the Available Bandwidth and CAC Using Fokker-Plank Equation

This section gives a estimation of the unused portion of the bandwidth using the diffusion approximation [7]. We introduce coordinate q on the ABR buffer. It corresponds to the number of cells. We assume that the number of cells in the ABR buffer takes a continuous value (real number). We assign $q = \widetilde{l}_1$ at the edge of the ABR buffer (fully occupied point) and $q = 0$ at the point far from $\widetilde{l}_1$ from the edge of the ABR buffer (Fig. 7).

We try to estimate, using the Fokker-Plank equation [10], the probability that heavy congestion occurs under congestion. The probability of which the queue length of the ABR buffer is in interval $[q, q+dq)$ at slot t is expressed as $P(q,t)\,dq$. We choose the initial state as $P(q,0) := \delta(q)$, where $\delta(\cdot)$ is Dirac δ function. We demand that $P(q,t)$ satisfies the following Fokker-Plank equation:

$$\begin{aligned}\frac{\partial}{\partial t}P(q,t) = \Big\{ &-\frac{\partial}{\partial q}A(q,t) \\ &+\frac{1}{2}\frac{\partial^2}{\partial q^2}B(q,t)\Big\}P(q,t).\end{aligned} \tag{10}$$

The first term on the right-hand side determines (deterministic) drift motion. The second term on the right-hand side describes the (stochastic) diffusion process. By the way, if $A(q,t) = 0$ and $B(q,t) =$ constant, Eq. (10) becomes the well-known diffusion equation. In this paper, we choose

$$A(q,t) := -(1-\rho^{\mathrm{a}}-\rho^{\mathrm{h}}),$$

and, from Eq. (6),

$$B(q,t) := \frac{\partial}{\partial t}\widetilde{V}(t) = \begin{cases} 2\beta t & 0 \leqq t < T_1, \\ 0 & T_1 \leqq t. \end{cases}$$

For simplicity, we do not adopt the boundary conditions in Eq. (10), corresponding to the capacity of ABR buffer, we can obtain the following solutions: Where $0 \leqq t < T_1$,

$$\begin{aligned}P(q,t) &= \frac{1}{\sqrt{2\pi\beta}\,t} \\ &\quad\times \exp\left\{-\frac{(q+(1-\rho^{\mathrm{a}}-\rho^{\mathrm{h}})\,t)^2}{2\beta\,t^2}\right\},\end{aligned} \tag{11}$$

Where $T_1 \leqq t$,

$$\begin{aligned}P(q,t) &= \frac{1}{\sqrt{2\pi\beta}\,T_1} \\ &\quad\times \exp\left\{-\frac{(q+(1-\rho^{\mathrm{a}}-\rho^{\mathrm{h}})\,t)^2}{2\beta\,T_1^2}\right\}.\end{aligned} \tag{12}$$

In both cases, $P(q,t)$ is the normal distribution with respect to q, when t is fixed.

Next, we show the framework for determining whether the available bandwidth for the ABR class is sufficient or not using solutions Eqs. (11) and (12).

Consider that we have a quality standard $\mathbf{P}$, which is the probability such that heavy congestion occurs in spite the fact that the cell emission rate of ABR connection j ($j = 1, 2, \ldots, m$) is limited to MCR M_j. We determine whether the available bandwidth is sufficient or not, by examining if the estimated probability is smaller or greater than $\mathbf{P}$.

We define ℓ_P, corresponding to the quality standard $\mathbf{P}$ using standard normal distribution, such that

$$\mathbf{P} = \frac{1}{\sqrt{2\pi}}\int_{\ell_P}^{\infty}\exp\left(-\frac{q^2}{2}\right)dq.$$

Consider the case in which congestion is detected, and the cell emission rate of all ABR connections, accommodated in the ABR buffer, becomes MCR at slot $t = 0$. Then we approximate the probability of the ABR buffer in heavy congestion at slot $t = T$ as

$$P_{\rm loss}(T) := \int_{\tilde{l}_1}^{\infty} P(q,T)\,dq, \tag{13}$$

and, in general, the probability that heavy congestion occurs under congestion conditions, is given by

$$\mathbf{P}_{\rm loss} := \max_{t \geq T_0} P_{\rm loss}(t).$$

In case of $T_1 < T_0$, we easily have

$$\mathbf{P}_{\rm loss} = P_{\rm loss}(T_0).$$

When solution Eq. (11) transforms into the standard normal distribution, the image of $\tilde{l}_1$ associated with the same transformation is given by

$$\tilde{l}_1 \to \frac{\tilde{l}_1 + (1-\rho^{\rm a}-\rho^{\rm h})\,t}{\sqrt{\beta}\,t}.$$

This image decreases with respect to an increasing t. Therefore, where $T_1 \geq T_0$, we have

$$\mathbf{P}_{\rm loss} = P_{\rm loss}(T_1).$$

Consequently, using $\widetilde{T} := \max(T_0, T_1)$, we have

$$\mathbf{P}_{\rm loss} = P_{\rm loss}(\widetilde{T}). \tag{14}$$

The actual procedure of ABR CAC used to determine whether $\mathbf{P}_{\rm loss}$ satisfies $\mathbf{P}$ or not is as follows:

1. When ABR connection j setup is requested, we transform $P(q,\widetilde{T})$, given by solutions Eqs. (11) or (12), to the standard normal distribution, and find the image of $\tilde{l}_1$, associated with the same transformation, $\tilde{\ell}_1$, as

$$\tilde{\ell}_1 := \frac{\tilde{l}_1 + (1-\rho^{\rm a}-\rho^{\rm h})\,\widetilde{T}}{\sqrt{\beta}\,T_1}. \tag{15}$$

2. We compare $\tilde{\ell}_1$ with ℓ_P. If $\tilde{\ell}_1 \geq \ell_P$, that is, $\mathbf{P}_{\rm loss} \leq \mathbf{P}$, we determine that the available bandwidth for the ABR class is sufficient and the connection setup request is accepted, and if $\tilde{\ell}_1 < \ell_P$, that is, $\mathbf{P}_{\rm loss} > \mathbf{P}$, we determine that the available bandwidth for the ABR class is insufficient and the network rejects the connection.

3.3 Simulation Results

This section shows a investigation the accuracy of above two estimations using simulation results.

The capacity of the transmission link is 150 Mbps. There are 10 high-priority connections and they are mutually independent. All of them have the same traffic descriptors and their PCR P_i and SCR S_i as 50 Mbps and 5 Mbps, respectively. In addition, all of them are in the periodic pattern of $x_i = 0$ shown in Sect. 2.2. A cell-buffer for high-priority connections has infinite cell places. There are 10 ABR connections and their MCRs are 1.5 Mbps. Therefore, total MCR is 15 Mbps. $\tilde{l}_1$ and $l_1^c + l_3$ in an ABR cell-buffer is 50 and 100 cell places, respectively.

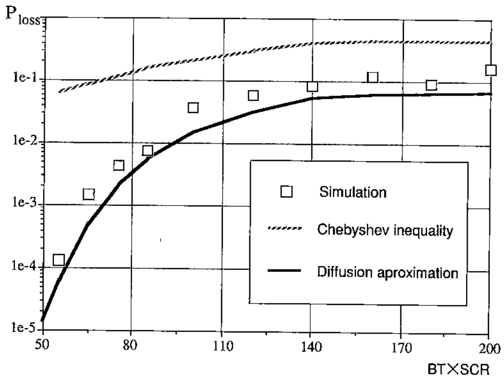

Fig. 8 Accuracy of estimating $\mathbf{P}_{\rm loss}$.

Figure 8 shows the probability $\mathbf{P}_{\rm loss}$ that heavy congestion occurs under congestion conditions with respect to BT of high-priority connections. The estimation using Chebyshev inequality results in exceeding overestimation of $\mathbf{P}_{\rm loss}$. The estimation using the diffusion approximation is more practical and accurate than that using Chebyshev inequality. Notice that the probability $\mathbf{P}_{\rm loss}$ is not Cell Loss Ratio (CLR). If our discussion is combined with the rate control mechanism, CLR is derived using $\mathbf{P}_{\rm loss}$. In general, $\mathbf{P}_{\rm loss}$ is greater than CLR standards. So our example gives around 10^{-3}.

Hereafter, we focus on the estimation using the diffusion approximation.

4. Design of Congestion Detecting Point on ABR Buffer

High-priority connections are setup or released independently of the ABR connection. Therefore, in spite of the fact that the available bandwidth for ABR is sufficient at that time, the bandwidth is not guaranteed after that time. In order to avoid this problem, we propose a method that dynamically changes the congestion detecting point in the ABR buffer. Before $\mathbf{P}_{\rm loss}$ is not satisfied, let l_1 in the ABR buffer increase. We show a scheme of changing the congestion detecting point in the ABR buffer corresponding to the diffusion approximation in Sect. 3.2.2.

From Eq. (13), when l_1 increases (of course $\tilde{l}_1$ increases simultaneously), $P_{\rm loss}(t)$ decreases; therefore, $\mathbf{P}_{\rm loss}$ decreases. In addition, since T_0 increases with respect to an increasing l_1, we can recognize that $\mathbf{P}_{\rm loss}$ decreases, from Eq. (14).

From another point of view, in spite of the fact that total capacity of ABR buffer, l $(= l_1 + l_2 + l_3)$, is fixed, excessive occupation of l_1 is not good. This is because congestion detection occurs frequently, and it restricts cell transmission. Since the main purpose of ABR is to support LAN services, frequent restriction of cell transmission reduces the appeal of ABR.

Therefore, whenever a high-priority connection is setup or released, it is desirable to choose the minimum l_1 such that $\mathbf{P}_{\rm loss}$ satisfies the standard $\mathbf{P}$. This corresponds to change $\tilde{l}_1$, dynamically, such that

$$\mathbf{P}_{\rm loss} = \mathbf{P}.$$

Since, this is equivalent to

$$\tilde{\ell}_1 = \ell_P,$$

from Eq. (15), we choose $\tilde{l}_1$ such that

$$\frac{\tilde{l}_1 + (1-\rho^{\rm a}-\rho^{\rm h})\,\widetilde{T}}{\sqrt{\beta}\,T_1} = \ell_P.$$

This is achieved in the following procedure whenever a high-priority connection is setup or released. When $\widetilde{T} = T_1$, since T_1 is independent of $\tilde{l}_1$, we can select an $\tilde{l}_1$ such that

$$\tilde{l}_1 = T_1\left(\sqrt{\beta}\,\ell_P - (1-\rho^{\rm a}-\rho^{\rm h})\right).$$

When $\widetilde{T} = T_0$, from Eq. (7), we can select an $\tilde{l}_1$ such that

$$\tilde{l}_1 = \frac{\rho^{\rm a}\,T_1\sqrt{\beta}\,\ell_P - (1-\rho^{\rm a}-\rho^{\rm h})}{1-\rho^{\rm h}}.$$

From the above procedure, we can guarantee that the standard $\mathbf{P}$ for ABR connections is independent of high-priority connections being setup or released.

5. Conclusion

This paper has described the fluctuation of link utilization, caused by high-priority connections, using Allan variance. Based on this, it has also shown the frameworks for estimating the available bandwidth and CAC for ABR, and have shown a design for the congestion detection point of the ABR buffer using the diffusion approximation.

To achieve a more accurate estimation of the unused portion of the bandwidth, we must take an appropriate boundary condition of the Fokker-Plank equation into account, and it is for further study.

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Appendix A: Proof of Theorem 1

Consider the following quantity;

$$\xi_i(t,\varsigma) := N_i(t,\varsigma) - E\left[N_i(t,\varsigma)\right].$$

From definition of ceiling patterns in Sect. 2.2, range of ξ_i is restricted as

$$|\xi_i(t,\varsigma)| < B_i S_i + 1.$$

Therefore,

$$E\left[\xi_i^2(t,\varsigma)\right] = O(1).$$

In addition, using $E\left[\xi_i(t,\varsigma)\right] = 0$, we have

$$\begin{aligned}\sigma_i^2(t,\varsigma) &= \frac{1}{t^2}\left\{E\left[N_i^2(t,\varsigma)\right] - E\left[N_i^2(t,\varsigma)\right]^2\right\}\\ &= \frac{1}{t^2}E\left[\xi_i^2(t,\varsigma)\right]\\ &= O(t^{-2}).\end{aligned}$$

□

Appendix B: Asymptotic Expression of $\sigma_i^2(t,\varsigma)$

Let high-priority connection i comply with GCRA, and it has periodic pattern specified in Sect. 2.2. We give a concrete expression of the Allan variance for link utilization, $\sigma_i^2(t,\varsigma)$ with respect to $\varsigma \in \Sigma_{({\rm p})i}$ in the asymptotic region $t \gg 1$. Here, we regard the number of cells as a continuous quantity, and adopt the following fluid model to cell flow. A high-priority connection i has two states. One is for cell emission at PCR P_i during $W_i^{\rm on}$ slots. The other is for cell emission at rate of x_i. These states periodically transit one after the other with period $W_i(x_i)$. This causes no problems in the asymptotic estimation of $\sigma_i^2(t,\varsigma)$.

First, for $\varsigma \in \Sigma_{({\rm p})i}$ having x_i, t is divided into two parts:

$$t = \kappa_i(x_i)W_i(x_i) + \tau_i(x_i),$$

where $\kappa_i(x_i)$ is a non-negative integer and $0 \leqq \tau_i(x_i) < W_i(x_i)$. We select an arbitrary time and set it to slot 0. Then, the minimum value of $N_i(t,\varsigma)$ is denoted as

$$\begin{aligned}N_i^{\min}(t,\varsigma) &= \kappa_i(x_i)\left(P_i W_i^{\rm on} + x_i W_i^{\rm off}(x_i)\right)\\ &\quad + x_i \min\left(\tau_i(x_i), W_i^{\rm off}(x_i)\right)\\ &\quad + P_i \max\left(0, (\tau_i(x_i) - W_i^{\rm off}(x_i))\right).\end{aligned}$$

The probability such that $N_i(t,\varsigma) = N_i^{\min}(t,\varsigma)$ is denoted as

$$\mathbf{P}_i^{\min} := \Pr[N_i = N_i^{\min}] = \frac{|\tau_i(x_i) - W_i^{\mathrm{off}}(x_i)|}{W_i(x_i)}.$$

Similarly, the maximum value of $N_i(t,\varsigma)$ is denoted as

$$\begin{aligned} N_i^{\max}(t,\varsigma) &= \kappa_i(x_i)\left(P_i W_i^{\mathrm{on}} + x_i W_i^{\mathrm{off}}(x_i)\right) \\ &\quad + P_i \min\left(\tau_i(x_i), W_i^{\mathrm{on}}\right) \\ &\quad + x_i \max\left(0, (\tau_i(x_i) - W_i^{\mathrm{on}})\right). \end{aligned}$$

The probability such that $N_i(t,\varsigma) = N_i^{\max}(t,\varsigma)$ is denoted as

$$\mathbf{P}_i^{\max} := \Pr[N_i = N_i^{\max}] = \frac{|\tau_i(x_i) - W_i^{\mathrm{on}}(x_i)|}{W_i(x_i)}.$$

Using the above notations, $\sigma_i^2(t,\varsigma)$ is denoted as

$$\begin{aligned} \sigma_i^2(t,\varsigma) &= \frac{1}{t^2}\left\{(N_i^{\max})^2 \mathbf{P}_i^{\max} + (N_i^{\min})^2 \mathbf{P}_i^{\min}\right\} \\ &\quad + \frac{1}{t^2}(1 - \mathbf{P}_i^{\max} - \mathbf{P}_i^{\min}) \\ &\quad \times \left\{\frac{(N_i^{\max} - N_i^{\min})^2}{3} + N_i^{\max} N_i^{\min}\right\} \\ &\quad - S_i^2. \end{aligned}$$

Here, we define

$$\Delta N_i(t,\varsigma) := N_i^{\max}(t,\varsigma) - N_i^{\min}(t,\varsigma),$$

and using

$$\begin{aligned} S_i &= \frac{1}{t}\left(N_i^{\max}\mathbf{P}_i^{\max} + N_i^{\min}\mathbf{P}_i^{\min}\right) \\ &\quad + \frac{1}{2t}\left(1 - \mathbf{P}_i^{\max} - \mathbf{P}_i^{\min}\right)(N_i^{\max} + N_i^{\min}), \end{aligned}$$

$\sigma_i^2(t,\varsigma)$ can be expressed as

$$\begin{aligned} \sigma_i^2(t,\varsigma) &= \frac{(\Delta N_i)^2}{t^2}\left\{\frac{1}{3}(1 + 2\mathbf{P}_i^{\max} - \mathbf{P}_i^{\min}) \right. \\ &\quad \left. - \frac{1}{4}(1 + \mathbf{P}_i^{\max} - \mathbf{P}_i^{\min})^2\right\}. \quad \text{(A· 1)} \end{aligned}$$

Appendix C: Proof of Theorem 2

For fixed x_i, that is, $\varsigma \in \Sigma_{(\mathrm{p})i}$, asymptotic expression of $\sigma_i^2(t,\varsigma)$ is given by Eq. (A· 1). For any t and x_i, $\Delta N_i(t,\varsigma)$ satisfies the following property:

$$\Delta N_i(t,\varsigma) \leqq P_i W_i^{\mathrm{on}}.$$

Then, for any x_i,

$$\sigma_i^2(t,\varsigma) < \frac{(\Delta N_i)^2}{t^2} \leqq \left(\frac{P_i W_i^{\mathrm{on}}}{t}\right)^2 \quad t \gg 1.$$

Therefore, we have

$$\sigma_{(\mathrm{p})i}^2(t) = \sup_{\varsigma \in \Sigma_{(\mathrm{p})i}} \sigma_i^2(t,\varsigma) < \left(\frac{P_i W_i^{\mathrm{on}}}{t}\right)^2 \quad t \gg 1.$$

From this and theorem 1, we have

$$\sigma_{(\mathrm{p})i}^2(t) = O(t^{-2}).$$

□

Appendix D: Proof of Theorem 3

Using ergodic property of $\varsigma \in \Sigma_i$, we have

$$\begin{aligned} \sigma_i^2(t,\varsigma) &= \frac{1}{t^2}E\left[\left(N_i(t,\varsigma) - S_i t\right)^2\right] \\ &= \frac{1}{t^2}\lim_{T\to\infty}\frac{1}{T}\int_0^T \left(N_i(\tau + t,\varsigma)\right. \\ &\quad \left. - N_i(\tau,\varsigma) - S_i t\right)^2 d\tau. \end{aligned}$$

For any $\varsigma \in \Sigma_i$, we can choose points $\{\tau_k;\ k = 0, 1, \ldots\}$ such that

$$\int_{\tau_k}^{\tau_{k+1}} \left(N_i(\tau + t,\varsigma) - N_i(\tau,\varsigma) - S_i t\right) d\tau = 0,$$

and $\tau_{k+1} - \tau_k \geqq W_i(0)$ for all k. Then we have

$$\begin{aligned} \sigma_i^2(t,\varsigma) &= \frac{1}{t^2}\lim_{K\to\infty}\frac{1}{\tau_K - \tau_0} \\ &\quad \times \sum_{k=0}^{K-1}\int_{\tau_k}^{\tau_{k+1}} \left(N_i(\tau + t,\varsigma)\right. \\ &\quad \left. - N_i(\tau,\varsigma) - S_i t\right)^2 d\tau. \end{aligned}$$

For any k, we can choose $\varsigma_k^* \in \Sigma_{(\mathrm{p})i}$ such that its period is $\tau_{k+1} - \tau_k$, and

$$\begin{aligned} &(\tau_{k+1} - \tau_k)E\left[\left(N_i(t_k^*,\varsigma_k^*) - S_i t_k^*\right)^2\right] \\ &\quad \geqq \int_{\tau_k}^{\tau_{k+1}} \left(N_i(\tau + t,\varsigma) - N_i(\tau,\varsigma) - S_i t\right)^2 d\tau, \end{aligned}$$

where, $t_k^* \leqq t$. From assumption 1,

$$\begin{aligned} &E\left[\left(N_i(t_k^*,\varsigma_k^*) - S_i t_k^*\right)^2\right] \\ &\quad \leqq \sup_{\varsigma^* \in \Sigma_{(\mathrm{p})i}} E\left[\left(N_i(t,\varsigma^*) - S_i t\right)^2\right]. \end{aligned}$$

Therefore,

$$\begin{aligned} \sigma_i^2(t,\varsigma) &\leqq \frac{1}{t^2}\lim_{K\to\infty}\frac{1}{\tau_K - \tau_0}\sum_{k=0}^{K-1}(\tau_{k+1} - \tau_k) \\ &\quad \times E\left[\left(N_i(t_k^*,\varsigma_k^*) - S_i t_k^*\right)^2\right] \\ &\leqq \frac{1}{t^2}E\left[\left(N_{(\mathrm{p})i}(t) - S_i t\right)^2\right] \\ &= \sigma_{(\mathrm{p})i}^2(t). \end{aligned}$$

Since it is valid for any $\varsigma \in \Sigma_i$, we have

$$\sigma_i^2(t) \leqq \sigma_{(\mathrm{p})i}^2(t).$$

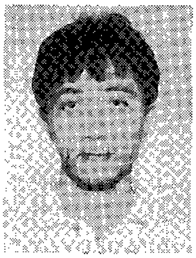
On the other hand, from Eqs. (2) and (3), it is obvious that

$$\sigma_i^2(t) \geqq \sigma_{(\mathrm{p})i}^2(t).$$

Therefore, we have

$$\sigma_i^2(t) = \sigma_{(\mathrm{p})i}^2(t).$$

□



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