PAPER

Traffic Contract Parameters and CAC Guaranteeing Cell-Loss Ratio in ATM Networks

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SUMMARY Connection Admission Control (CAC) is a key part of traffic control and still leaves several challenging problems peculiar to ATM networks. One of these problems is how to assign sufficient bandwidth for any cell arrival process that satisfies the source traffic descriptor values specified by negotiation between the network and a user at the connection setup. Because the source traffic descriptor cannot describe the actual source traffic characteristics completely, it has already been studied extensively that how to estimate sufficient bandwidth under the assumption that the actual traffic parameter values in the source traffic descriptor are equal to the negotiated values. This paper extends the studies in the literature to how to estimate sufficient bandwidth only assuming that the actual values satisfy the negotiated values, that is the actual values is less than or equal to the negotiated values. We show the sufficient condition for negotiated source traffic descriptors ensuring that the cell-loss ratio calculated from the negotiated values is always the upper-bound of the actual cell-loss ratio. Using this condition, we propose a CAC that can guarantee cell-loss ratio objective so far as a user satisfies the source traffic descriptor values.

key words: ATM, CAC, cell-loss ratio, nonparametric approach

1. Introduction

The Asynchronous Transfer Mode (ATM) is a key technology for Broadband Integrated Services Digital Networks (B-ISDN) [1]. ATM networks must support many kinds of traffic accompanied by a variety of services and they must guarantee the quality of service (QoS). Traffic control is therefore a key technology to attain such requirements. Connection Admission Control (CAC) is a particularly important control method and it is an issue peculiar to ATM networks.

The principle of CAC is as follows: At the time of connection setup, a user specifies the QoS requirements and the anticipated traffic characteristics by using a source traffic descriptor. A network bases the assignment of resources for the connection on the source traffic descriptor values and the QoS requirements, and the connection is rejected if there are not enough network resources available. In this paper, we assume that the source traffic descriptor consists of the peak cell rate and the average cell rate. If the connection is admitted by CAC, the network monitors the peak cell rate and the average cell rate of the connection to check whether they are less than or equal to the negotiated one.

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CAC still leaves several challenging problems for ATM networks. One of them is how to assign sufficient bandwidth for any cell arrival processes that satisfy the source traffic descriptor values. These values are specified by negotiation between the network and users at the time of connection setup, but the actual source traffic values are usually less than those negotiated because a user is required to satisfy the some traffic descriptor. Therefore, there are two aspects to solving this bandwidth-assignment problem:

(i) how to estimate sufficient for any cell arrival process bandwidth under the assumption that the negotiated values are equal to the actual values; and

(ii) how to estimate sufficient bandwidth without assuming that the negotiated and actual values are equal.

All the CAC algorithms in previous works [2]–[5] implicitly assume that the negotiated values and the actual values are the same. Therefore, under the assumption, the works give the sufficient bandwidth for any cell arrival process, that is, they give the solutions for the above problem (i). In general, the assumption is not always correct. So here we need to consider the second aspect (the above problem (ii)) and its integration with one of these CAC algorithms.

We can show that the second aspect is essential by using the following simple example. To begin with, we assume the following for virtual paths (VPs) to simplify the discussion. There are one or more VPs accommodated in a transmission link and a VP (connection) consists of VP links (Fig. 1) [2]. Each VP link is assumed to

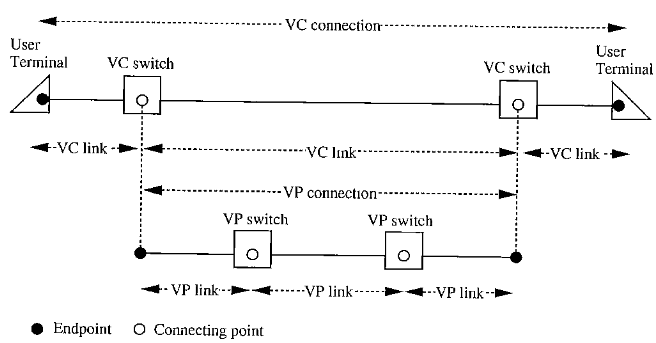

Fig. 1 Hierarchical relationship in the ATM layer.

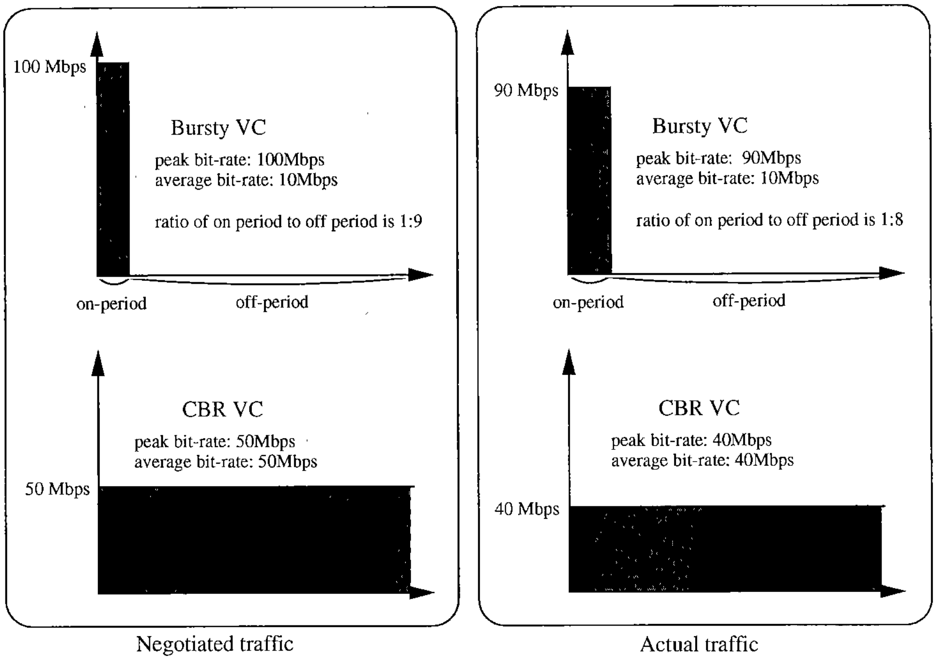

Fig. 2 Example of virtual channels (VCs).

have a rigid boundary through VP shaping [2], in other words, it is guaranteed that each VP link is assigned a fixed bandwidth. That is, we assume no statistical multiplexing among VP links. As a result, we can assume that each VP (connection) has a fixed bandwidth.

Consider a VP whose output buffer size is finite and whose bandwidth is 50 Mbps. There are two virtual channels (VCs) in this VP: one has a negotiated peak bit-rate of 100 Mbps and a negotiated average bit-rate of 10 Mbps (Fig. 2); the other has a negotiated peak bit-rate of 50 Mbps and a negotiated average bit-rate of 50 Mbps (that is, the CBR VC). When the burst is very long, the overall cell-loss ratio of the VP is at most $1/6$ when the negotiated values are identical to the actual value. But if the first VC has an actual peak bit-rate of 90 Mbps and an actual average bit-rate of 10 Mbps, and the second VC has an actual peak bit-rate of 40 Mbps and an actual average bit-rate of 40 Mbps, the overall cell-loss ratio of the VP is $8/45$ ($>1/6$). Similar problem is also occurred in the cell-loss ratio of individual connection. The cell-loss ratio of the second VC using the negotiated or actual cell rates is $1/15$ or $8/117$ ($>1/15$), respectively. Therefore, evaluating the upper-bound cell-loss ratio by using the negotiated values therefore does not always give the actual upper-bound.

This is because that cell-loss ratio is ratio of the number of lost cells to the number of incoming cells, and both of the numerator and the denominator decrease when the number of incoming cells decrease. So we do not know the cell-loss ratio increase or decrease when the number of incoming cells decrease.

This implies that QoS objective cannot be satisfied because of the insufficient bandwidth if you do not take account of the fact that the negotiated values are not same as the actual values.

This paper shows the sufficient condition on the negotiated source traffic descriptors that guarantees that the cell-loss ratio evaluated using negotiated values is always the upper-bound of the actual cell-loss ratio. This paper also combines aspects (i) and (ii) and proposes a CAC that includes this condition.

2. CAC Using a Nonparametric Approach

For simplification, we assume that a VP consists of VCs that have the same cell-loss ratio objective and have similar (but not the same, in general) traffic characteristics. We define the cell-loss ratio of a VP as the cell-loss ratio for all the cells belonging to all the VCs in the VP. We assume CAC decision is performed using the cell-loss ratio of the VP for simple CAC calculation. If we adopt CAC decision using the cell-loss ratio of individual VCs, it needs heavy calculation because we must re-calculate the cell-loss ratio of all individual VCs at the time of connection setup. In addition, it also cannot avoid the problem shown in Introduction.

A nonparametric approach [2] to CAC has been proposed for assigning bandwidth for any cell stream under the condition that the specified values of a source traffic descriptor are identical to the actual values. The decisions of this CAC are based on the upper bound of the cell-loss ratio derived from the source traffic descriptors without assuming a particular parametric model for the cell arrival process. This paper assumes this CAC and extends it to guarantee the bandwidth is large enough for any cell stream whose specified values of a source traffic descriptor are greater than or equal to the actual values; that is, for any cell stream conforming to the specification of a source traffic descriptor.

The non-parametric approach can be described as follows. Time is divided into fixed-length slots and a single slot corresponds to the transmission time of an ATM cell. The slot length is defined as L/C, where L is the cell length and C is the capacity of the VP. The cell-loss ratio B_{cell} of the VP can be upper-bounded by using the probability $p(k)$ of k cells arriving during a fixed interval equal to γ slots:

$$B_{cell} \leqq \frac{\sum_k [k-\gamma]^+ p(k)}{\sum_k kp(k)}, \tag{1}$$

where

$$[x]^+ = \begin{cases} x & x \geqq 0, \\ 0 & x < 0. \end{cases}$$

When the output buffer size is K, we choose $\gamma = K+1$ [6]. Regardless of the cell arrival process model, (1) is valid if a stationary distribution $\{p(k), k = 0, 1, \ldots\}$ exists. We consider n VCs in the VP and denote a set of VCs

$$\mathrm{N}_n \equiv \{1, 2, \ldots, n\}.$$

Knowing the probability $\{p_i(k), k = 0, 1, \ldots\} (i = 1, \ldots, n)$ of the number of cells arriving from each VC during an interval of γ slots and using the fact that $p(k) = \prod^{\star}_{i \in \mathrm{N}_n} p_i(k)$, we get

$$\begin{aligned} B_{cell} &\leqq B(p_1, \ldots, p_n; \gamma) \\ &\equiv \frac{\sum_k [k-\gamma]^+ \prod^{\star}_{i \in \mathrm{N}_n} p_i(k)}{\sum_k k \prod^{\star}_{i \in \mathrm{N}_n} p_i(k)}, \end{aligned}$$

where, $\prod^{\star}$ represents convolution and, for example, $\prod^{\star}_{i=1,2} p_i(k) = \sum_{j=0}^{n} p_1(j) p_2(k-j)$. But because it is difficult to know $p_i(k)$, we use $\varphi_i(k)$ instead of $p_i(k)$. For the i-th VC, $\varphi_i(k)$ is derived from the true maximum number of cells arriving during the fixed time period γ (MNA) and the true average number of arriving cells during the fixed time period γ (ANA). Let R_i be the MNA, A_i be the ANA, and define $\varphi_i(k)$ as

$$\varphi_i(k) = \begin{cases} 1 - A_i/R_i & k = 0, \\ A_i/R_i & k = R_i, \\ 0 & \text{otherwise.} \end{cases}$$

Using $\varphi_i(k), (i = 1, \ldots, n)$, we can get the upper-bound cell-loss formula [2],

$$\begin{aligned} B_{cell} &\leqq B(\varphi_1, \varphi_2, \ldots, \varphi_n; \gamma) \\ &= \frac{\sum_k [k-\gamma]^+ \prod^{\star}_{i \in \mathrm{N}_n} \varphi_i(k)}{\sum_k k \prod^{\star}_{i \in \mathrm{N}_n} \varphi_i(k)}. \end{aligned}$$

We cannot get the true MNA and ANA at the connection setup, but we can use the specified MNA and ANA. Assume, for the i-th VC, that the traffic descriptors specified by the user when the connection is set up are given as peak cell rate $\widehat{R}_i$ and average cell rate $\widehat{A}_i$. We can provide the MNA $\widetilde{R}_i$ and the ANA $\widetilde{A}_i$ values based on the negotiated traffic descriptors as

$$\widetilde{R}_i = \lceil \gamma L \widehat{R}_i / C \rceil, \ \widetilde{A}_i = \gamma L \widehat{A}_i / C, \tag{2}$$

where $\lceil x \rceil$ represents the smallest integer greater than or equal to x.

We assume that there are no cell delay variations (CDV). Equations similar to (2) are, however, applicable when there is CDV [7], and the following discussion is valid then. So to simplify the explanation, we do not treat CDV in this paper.

We define the following quantity:

$$B(\widetilde{\theta}_1, \widetilde{\theta}_2, \ldots, \widetilde{\theta}_n; \gamma) \equiv \frac{\sum_k [k-\gamma]^+ \prod^{\star}_{i \in \mathrm{N}_n} \widetilde{\theta}_i(k)}{\sum_k k \prod^{\star}_{i \in \mathrm{N}_n} \widetilde{\theta}_i(k)}, \tag{3}$$

where, for the i-th VC,

$$\widetilde{\theta}_i(k) = \begin{cases} 1 - \widetilde{A}_i/\widetilde{R}_i & k = 0, \\ \widetilde{A}_i/\widetilde{R}_i & k = \widetilde{R}_i, \\ 0 & \text{otherwise.} \end{cases}$$

The relationships between the true MNA R_i and ANA A_i and the negotiated MNA $\widetilde{R}_i$ and ANA $\widetilde{A}_i$ are

$$\begin{cases} 1 \leqq R_i \leqq \widetilde{R}_i, \\ 0 < A_i \leqq \widetilde{A}_i. \end{cases}$$

3. Bandwidth Assignment Model

The CAC function needs to decide, at the connection setup and according to traffic descriptors, whether the request for establishing a connection should be accepted. In practice, however, the difference between actual traffic characteristics and the traffic characteristics described by a source traffic descriptor prevent us from reaching a correct decision (see the example given in the Introduction). Based on the CAC using the non-parametric approach, the problem is described as

$$B_{cell} \nleqq B(\widetilde{\theta}_1, \widetilde{\theta}_2, \ldots, \widetilde{\theta}_n; \gamma),$$

if $R_i \neq \widetilde{R}_i$ or $A_i \neq \widetilde{A}_i$. There are two causes of this problem: one is the difference between the actual MNA and the negotiated MNA, and the other is the difference between the actual ANA and the negotiated ANA. We can remove the first cause by using the result shown in Ref. [6]. We define

$$\theta_i(k; r_i, a_i) \equiv \begin{cases} 1 - a_i/r_i & k = 0, \\ a_i/r_i & k = r_i, \\ 0 & \text{otherwise.} \end{cases}$$

The function $B(\theta_1,\ldots,\theta_n;\gamma)$ is then an increasing function of r_i. Therefore,

$$B(\theta_1(R_1,a_1),\theta_2(R_2,a_2),\ldots,\theta_n(R_n,a_n);\gamma) \leqq B(\theta_1(\widetilde{R}_1,a_1),\theta_2(\widetilde{R}_2,a_2),\ldots,\theta_n(\widetilde{R}_n,a_n);\gamma).$$

In particular, when $a_i = A_i$ for all i,

$$B_{cell} \leqq B(\theta_1(\widetilde{R}_1,A_1)\ldots,\theta_n(\widetilde{R}_n,A_n);\gamma). \tag{4}$$

In other words, the use of the negotiated MNA as the actual MNA provides the largest evaluation of cell-loss ratio for any ANA. The remainder of this paper can therefore focus on the later factor — that is the difference between the actual ANA and the negotiated ANA under the assumption that the true MNA is the negotiated one — in order to find a way of assigning bandwidth large enough for any cell stream conforming to the descriptors.

Hereafter we redefine $\theta_i(k;a_i) \equiv \theta_i(k;\widetilde{R}_i,a_i)$. Therefore $\widetilde{\theta}_i(k) = \theta_i(k;\widetilde{A}_i)$.

4. Relations between the Negotiated ANA and Cell-Loss Ratio

Consider a set S of VCs. When $S \subseteqq \mathbf{N}_n$ we define the set function $\mathcal{B}_\gamma(S)$ on $\mathbf{N}_n$ as follows:

$$\mathcal{B}_\gamma(S) \equiv B(q_1^S,q_2^S,\ldots,q_n^S;\gamma) = \frac{\sum_k [k-\gamma]^+ \prod^\star_{i\in S} \widetilde{\theta}_i(k)}{\sum_k k \prod^\star_{i\in S} \widetilde{\theta}_i(k)} \qquad S \neq \emptyset,$$

$$\mathcal{B}_\gamma(\emptyset) \equiv 0,$$

where we define

$$q_i^S(k) \equiv \begin{cases} \widetilde{\theta}_i(k) & i \in S, \\ \mathbf{1}(k) & \text{otherwise.} \end{cases}$$

We also define

$$\mathbf{1}(k) \equiv \begin{cases} 1 & k = 0, \\ 0 & \text{otherwise.} \end{cases}$$

Lemma 1: If we assume that $\{f(k), k = 0,1,\ldots\}$ is an arbitrary probability distribution such that $f(k) \neq \mathbf{1}(k)$, then

$$B(f,\theta_i(A_i);\gamma) \leqq \max\left\{B(f,\widetilde{\theta}_i;\gamma), B(f,\mathbf{1};\gamma)\right\},$$

and if $f(k) = \mathbf{1}(k)$,

$$B(f,\theta_i(A_i);\gamma) = B(f,\widetilde{\theta}_i;\gamma).$$

Proof
See Appendix A.1.

Theorem 1: There is a set $S(\subseteqq \mathbf{N}_n)$ such that

$$B(\theta_1(A_1),\theta_2(A_2),\ldots,\theta_n(A_n);\gamma) \leqq \mathcal{B}_\gamma(S).$$

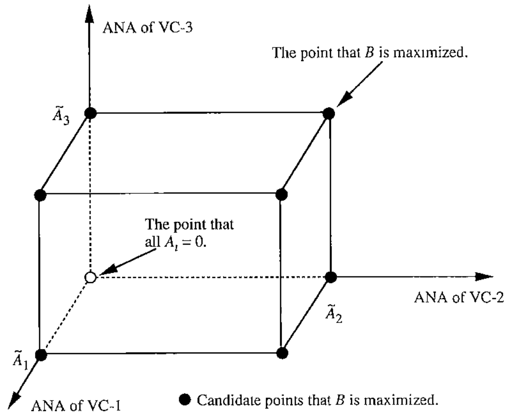

Fig. 3 Example of the point that B is maximized.

Proof
See Appendix A.2.

The physical meaning of this theorem can be seen by considering $B(\theta_1,\ldots,\theta_n;\gamma)$, which is a function of A_i, under fixed $\widetilde{R}_i$. The range of this variable A_i is $(0,\widetilde{A}_i]$ and forms an n-dimensional rectangular domain. Therefore *Theorem* 1 shows that the function is maximized at the end-points of the range or at a certain vertex of the rectangular domain, except one in which all A_i are zero (Fig. 3).

5. Conditions for Specifying Traffic Contract Parameters

In this section, we show the sufficient condition providing the semi-order relation on set $\mathbf{N}_n$. *Theorem* 1 concludes that $B(\theta_i(k;a_i);\gamma)$ can be maximized at $a_i = 0$ or A_i, and this relation enables us to avoid searching the $2^n - 1$ end-points of A_is for those that maximize the function $B(\theta_1,\theta_2,\ldots,\theta_n;\gamma)$. Thus, we can easily and naturally evaluate the cell-loss ratio.

First, we define the set functions $\mathcal{F}_\gamma(S)$ and $\mathcal{A}(S)$ on $\mathbf{N}_n$. When $S \subseteqq \mathbf{N}_n$,

$$\mathcal{F}_\gamma(S) \equiv \sum_{k=\gamma}^{\infty} \prod_{i\in S}^{\star} \widetilde{\theta}_i(k),$$

$$\mathcal{A}(S) \equiv \sum_{i\in S} \widetilde{A}_i.$$

The function $\mathcal{F}_\gamma(S)$ means the probability that the number of cells arriving within an interval of γ slots is greater than or equal to γ. And the function $\mathcal{A}(S)$ is the average number of cells arriving within an interval of γ slots.

Lemma 2: For any $S \subseteqq \mathbf{N}_n$ and

$${}^\forall i \in \mathbf{N}_n \backslash S,$$

we have

$$\mathcal{F}_\gamma(S) \leqq \left(\frac{\mathcal{A}(S)}{\widetilde{R}_i}\right)\left(\mathcal{B}_{\gamma-\widetilde{R}_i}(S) - \mathcal{B}_\gamma(S)\right)$$

$$\leqq \mathcal{F}_{\gamma-\widetilde{R}_i}(S).$$

Proof
See Appendix B.1.

This shows that the set function $\mathcal{F}_\gamma(S)$ is nonincreasing with respect to γ.

Lemma 3: If there is $S\ (\subseteqq \mathbf{N}_n)$ such that

$$\mathcal{B}_\gamma(S) \leqq \mathcal{F}_\gamma(S),$$

then for any $i \in \mathbf{N}_n$ we have

$$\mathcal{B}_\gamma(S) \leqq \mathcal{B}_\gamma(S^+).$$

where

$$S^+ \equiv S \cup \{i\}.$$

Proof
See Appendix B.2.

Theorem 2: If at least $n-1$ VCs in $\mathbf{N}_n$ satisfy the inequality

$$\frac{\widetilde{R}_i^2}{\widetilde{A}_i} \leqq \gamma, \tag{5}$$

then, in terms of two sets of VCs

$${}^\forall S_1, S_2 \subseteqq \mathbf{N}_n,$$

we have

$$S_1 \subseteqq S_2 \Longrightarrow \mathcal{B}_\gamma(S_1) \leqq \mathcal{B}_\gamma(S_2). \tag{6}$$

Proof
See Appendix B.3.

According to *Theorems* 1 and 2, if the negotiated parameters specified by users of, at least, $n-1$ VCs satisfy condition (5), then we have the following equality:

$$\max_{S \subseteqq \mathbf{N}_n} \mathcal{B}_\gamma(S) = \mathcal{B}_\gamma(\mathbf{N}_n).$$

Combined this with inequality (4), we have

$$\begin{aligned} &B(\varphi_1, \varphi_2, \ldots, \varphi_n; \gamma) \\ &\quad \leqq B(\theta_1(A_1), \theta_2(A_2), \ldots, \theta_n(A_n); \gamma) \\ &\quad \leqq B(\widetilde{\theta}_1, \widetilde{\theta}_2, \ldots, \widetilde{\theta}_n; \gamma). \end{aligned}$$

Therefore it is guaranteed that the cell-loss ratio evaluated from the negotiated MNA and ANA is the upper-bound of the actual cell-loss ratio. That is,

$$B_{cell} \leqq B(\widetilde{\theta}_1, \widetilde{\theta}_2, \ldots, \widetilde{\theta}_n; \gamma).$$

6. CAC Guaranteeing Cell-Loss Ratio

Here we describe a CAC that includes condition (5). We assume that n VCs have already been accepted in the VP. At the new VC setup, if for at least n VCs including the new VC, the negotiated MNA and ANA satisfy condition (5), we evaluate cell-loss ratio of VP by using (3) and decide whether to accept or reject based on that calculated (3) is less than the cell-loss ratio objective. Otherwise, if there are m VCs ($m \leqq n+1$) which do not satisfy condition (5), we take the following new negotiated ANA $\bar{A}_i$ for the m VCs:

$$\bar{A}_i = \frac{\widetilde{R}_i^2}{\gamma}. \tag{7}$$

For the m VCs, we use $\bar{A}_i$ substituted for $\widetilde{A}_i$. Then for n VCs of all VCs, condition (5) is satisfied. We also use (3) to evaluate the cell-loss ratio of VP from the new negotiated parameters and decide whether to accept or reject.

Note that the new $\bar{A}_i$ calculated by using (7) is greater than $\widetilde{A}_i$. Therefore we give the VC extra bandwidth.

7. Numerical Example

Figure 4 shows examples of behavior of cell-loss ratio with respect to actual ANA of a VC. Case A in Fig. 4 means the cell-loss ratio evaluated using (3) does not upper-bound the actual cell-loss ratio of the VP. In otherwise, case B has no such problem. Here we give an example of case A which gives more realistic than that given in the Introduction. We assume that the capacity of the VP is 156 Mbps, that the output buffer size is 127 cell places, and that there are two types of VCs in the VP. The negotiated values for type-1 VCs are a peak bit rate of 1.5 Mbps and an average bit rate of 0.3 Mbps. The negotiated values for type-2 VCs are a peak bit rate of 95.0 Mbps and an average bit rate of 0.6 Mbps. Type-1 VCs satisfy (5), but type-2 VCs do not.

If there are 100 type-1 VCs and 2 type-2 VCs in the VP and if the actual average bit rate of type-1 VCs is less than the negotiated bit-rate, the evaluated cell-loss ratio is not the upper-bound of the actual cell-loss ratio of the VP. Figure 5 shows that under these conditions the actual cell-loss ratio is greater than that evaluated from the negotiated values, but smaller than that evaluated by using (7).

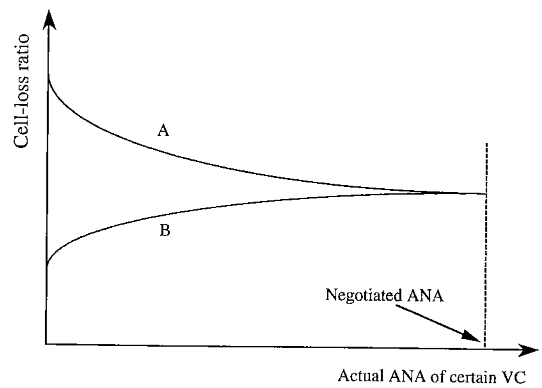

Fig. 4 Behavior of cell-loss ratio.

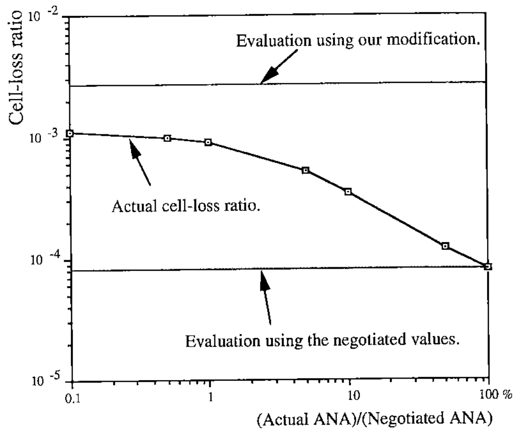

Fig. 5 Relationship between cell-loss ratio and actual ANA (type-1 VCs).

8. Conclusions

We derived the sufficient condition under which the cell-loss ratio derived, using the nonparametric approach, from the negotiated MNA and ANA is guaranteed to be the upper-bound of the actual cell-loss ratio of the VP. Because this condition can be imposed on an individual VC, a CAC that satisfies this condition can be developed by simply converting the negotiated traffic descriptors into the traffic parameters for the CAC.

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Appendix A: Proof of Theorem 1

A.1: Proof of lemma 1

The partial derivative $B(f,\theta_i(A_i);\gamma)$ with respect to A_i is

$$\frac{\partial B(f,\theta_i(A_i);\gamma)}{\partial A_i} = \frac{1}{(\mathrm{E}_f+A_i)^2}\left[\frac{\mathrm{E}_f}{\widetilde{R}_i}\sum_k\{[k-\gamma+\widetilde{R}_i]^+ - [k-\gamma]^+\}f(k) - \sum_k[k-\gamma]^+ f(k)\right],$$

where $\mathrm{E}_f = \sum_{k=0}^{\infty} k\,f(k)$. We conclude that the sign of $\partial B(f,\theta_i(A_i);\gamma)/\partial A_i$ is independent of A_i. Therefore, we have

$$B(f,\theta_i(A_i);\gamma) \leqq \begin{cases} B(f,\widetilde{\theta}_i;\gamma) & \partial B(f,\theta_i(A_i);\gamma)/\partial A_i \geqq 0, \\ B(f,\mathbf{1};\gamma) & \partial B(f,\theta_i(A_i);\gamma)/\partial A_i < 0. \end{cases}$$

If $f(k) = \mathbf{1}(k)$, then

$$\frac{\partial B(f,\theta_i(A_i);\gamma)}{\partial A_i} = \frac{\partial}{\partial A_i}\frac{[\widetilde{R}_i-\gamma]^+}{\widetilde{R}_i} = 0$$

and

$$B(f,\theta_i(A_i);\gamma) = B(f,\widetilde{\theta}_i;\gamma). \qquad \square$$

A.2: Proof of theorem 1

We define

$$f_1(k) \equiv [\theta_2(A_2)\star\theta_3(A_3)\star\cdots\star\theta_n(A_n)](k),$$

where binomial operator $\star$ represents convolution. Next we can define

$$f_2(k) \equiv [q_1\star\theta_3(A_3)\star\cdots\star\theta_n(A_n)](k)$$

$$\vdots$$

$$f_i(k) \equiv [q_1\star\cdots\star q_{i-1}\star\theta_{i+1}(A_{i+1})\star\cdots\cdots\star\theta_n(A_n)](k)$$

$$\vdots$$

$$f_n(k) \equiv [q_1\star q_2\star\cdots\star q_{n-1}](k)$$

by using the following relation

$$[f_{i+1}\star\theta_{i+1}(A_{i+1})](k) = [f_i\star q_i](k), \qquad \text{(A$\cdot$1)}$$

where

$$q_i(k) \equiv \begin{cases} \widetilde{\theta}_i(k) & \partial B(f_i,\theta_i(A_i))/\partial A_i \geqq 0, \\ \mathbf{1}(k) & \partial B(f_i,\theta_i(A_i))/\partial A_i < 0. \end{cases}$$

Using *Lemma* 1 and (A$\cdot$1), we have

$$\begin{aligned} &B(\theta_1(A_1),\ldots,\theta_n(A_n)) \\ &\quad = B(f_1,\theta_1(A_1)) \\ &\quad \leqq B(f_1,q_1) = B(f_2,\theta_2(A_2)) \\ &\quad \vdots \\ &\quad \leqq B(f_{n-1},q_{n-1}) = B(f_n,\theta_n(A_n)) \\ &\quad \leqq B(f_n,q_n) = B(q_1,\ldots,q_n), \end{aligned}$$

where

$$[q_1\star q_2\star\cdots\star q_n](k) \neq \mathbf{1}(k).$$

We conclude that there is $S \subseteqq \mathbf{N}_n$ such that

$$B(\theta_1(A_1),\ldots,\theta_n(A_n)) \leqq B(q_1,\ldots,q_n) = \mathcal{B}_\gamma(S). \qquad \square$$

Appendix B: Proof of Theorem 2

B.1: Proof of lemma 2

We have

$$\begin{aligned}
&\mathcal{B}_{\gamma-\widetilde{R}_i}(S)-\mathcal{B}_\gamma(S)\\
&=\frac{1}{\mathcal{A}(S)}\sum_k\left\{[k-\gamma+\widetilde{R}_i]^+-[k-\gamma]^+\right\}\prod_{j\in S}{}^{\star}\widetilde{\theta}_j(k)\\
&=\frac{1}{\mathcal{A}(S)}\left\{\widetilde{R}_i\sum_{k=\gamma}^{\infty}\prod_{j\in S}{}^{\star}\widetilde{\theta}_j(k)\right.\\
&\quad\left.+\sum_{k=\gamma-\widetilde{R}_i}^{\gamma-1}(k-\gamma+\widetilde{R}_i)\prod_{j\in S}{}^{\star}\widetilde{\theta}_j(k)\right\}\\
&\geqq\frac{\widetilde{R}_i}{\mathcal{A}(S)}\mathcal{F}_\gamma(S),
\end{aligned}$$

and

$$\begin{aligned}
&\mathcal{B}_{\gamma-\widetilde{R}_i}(S)-\mathcal{B}_\gamma(S)\\
&=\frac{1}{\mathcal{A}(S)}\left\{\widetilde{R}_i\sum_{k=\gamma-\widetilde{R}_i}^{\infty}\prod_{j\in S}{}^{\star}\widetilde{\theta}_j(k)\right.\\
&\quad\left.+\sum_{k=\gamma-\widetilde{R}_i}^{\gamma-1}(k-\gamma)\prod_{j\in S}{}^{\star}\widetilde{\theta}_j(k)\right\}\\
&\leqq\frac{\widetilde{R}_i}{\mathcal{A}(S)}\mathcal{F}_{\gamma-\widetilde{R}_i}(S).
\end{aligned}$$

□

B.2: Proof of lemma 3

If $i\in S$, then $S=S^+$ and we have

$$\mathcal{B}_\gamma(S)=\mathcal{B}_\gamma(S^+).$$

If $i\notin S$, we have

$$\begin{aligned}
\mathcal{B}_\gamma(S^+)&=\frac{1}{\mathcal{A}(S^+)}\sum_k[k-\gamma]^+\prod_{j\in S^+}{}^{\star}\widetilde{\theta}_j(k)\\
&=\frac{\mathcal{A}(S)}{\mathcal{A}(S^+)}\left\{\mathcal{B}_\gamma(S)\right.\\
&\quad\left.+(\widetilde{A}_i/\widetilde{R}_i)(\mathcal{B}_{\gamma-\widetilde{R}_i}(S)-\mathcal{B}_\gamma(S))\right\}.
\end{aligned}$$

Therefore $\mathcal{B}_\gamma(S^+)\geqq\mathcal{B}_\gamma(S)$ is equivalent to

$$\left(\frac{\mathcal{A}(S)}{\widetilde{R}_i}\right)\left(\mathcal{B}_{\gamma-\widetilde{R}_i}(S)-\mathcal{B}_\gamma(S)\right)\geqq\mathcal{B}_\gamma(S).$$

From the assumption in *Lemma* 3 and from *Lemma* 2, we can show that

$$\left(\frac{\mathcal{A}(S)}{\widetilde{R}_i}\right)\left(\mathcal{B}_{\gamma-\widetilde{R}_i}(S)-\mathcal{B}_\gamma(S)\right)\geqq\mathcal{F}_\gamma(S)\geqq\mathcal{B}_\gamma(S).$$

□

B.3: Proof of theorem 2

We define

$$S^-\equiv S\backslash i$$

in terms of $S\subseteqq\mathbf{N}_n,\ i\in S$. Consider the sufficient condition that if

$$\mathcal{B}_\gamma(S^-)\leqq\mathcal{F}_\gamma(S^-),$$

then its relation is conserved in terms of S. That is,

$$\mathcal{B}_\gamma(S)\leqq\mathcal{F}_\gamma(S).$$

Using *Lemma* 2, we have

$$\begin{aligned}
&\mathcal{F}_\gamma(S)-\mathcal{B}_\gamma(S)\\
&\quad=\mathcal{F}_\gamma(S^-)+\frac{\widetilde{A}_i}{\widetilde{R}_i}\left(\mathcal{F}_{\gamma-\widetilde{R}_i}(S^-)-\mathcal{F}_\gamma(S^-)\right)\\
&\quad-\frac{\mathcal{A}(S^-)}{\mathcal{A}(S)}\left\{\mathcal{B}_\gamma(S^-)+\frac{\widetilde{A}_i}{\widetilde{R}_i}(\mathcal{B}_{\gamma-\widetilde{R}_i}(S^-)-\mathcal{B}_\gamma(S^-))\right\}\\
&\quad\geqq\widetilde{A}_i\left(\frac{1}{\widetilde{R}_i}-\frac{1}{\mathcal{A}(S)}\right)\left(\mathcal{F}_{\gamma-\widetilde{R}_i}(S^-)-\mathcal{F}_\gamma(S^-)\right).
\end{aligned}$$

Therefore

$$\mathcal{A}(S)\geqq\widetilde{R}_i \tag{A$\cdot$2}$$

is the sufficient condition for $\mathcal{F}_\gamma(S)\geqq\mathcal{B}_\gamma(S)$. Next we choose $j_l\in S\ (l=1,\ldots,s-1)$ for arbitrary $S\subseteqq\mathbf{N}_n$ ($|S|=s$) as follows:

$$\widetilde{R}_{j_1}\geqq\widetilde{R}_{j_2}\geqq\cdots\geqq\widetilde{R}_{j_{s-1}} \tag{A$\cdot$3}$$

and, for $l=1,2,\ldots,s-1$,

$$\frac{\widetilde{R}_{j_l}^2}{\widetilde{A}_{j_l}}\leqq\gamma. \tag{A$\cdot$4}$$

In addition, we define $S_{j_m}\subseteqq S$ as follows:

$$\begin{aligned}
S_{j_0}&\equiv\emptyset,\\
S_{j_1}&\equiv\{j_1\},\\
S_{j_2}&\equiv\{j_1,j_2\},\\
&\ \vdots\\
S_{j_m}&\equiv S_{j_{m-1}}\cup\{j_m\},\\
&\ \vdots\\
S_{j_s}&\equiv S.
\end{aligned}$$

In terms of S_{j_0}, we have

$$\mathcal{B}_\gamma(S_{j_0})=\mathcal{F}_\gamma(S_{j_0}).$$

If for some S_{j_m} $(m\leqq s-2)$,

$$\mathcal{B}_\gamma(S_{j_m})\leqq\mathcal{F}_\gamma(S_{j_m}),$$

and

$$\sum_{l=1}^{m+1} \widetilde{R}_{j_l} < \gamma,$$

then we easily have

$$\mathcal{B}_\gamma(S_{j_{m+1}}) = \mathcal{F}_\gamma(S_{j_{m+1}}).$$

On the other hand, if

$$\sum_{l=1}^{m+1} \widetilde{R}_{j_l} \geqq \gamma, \tag{A$\cdot$5}$$

from the relations (A$\cdot$4) and (A$\cdot$3), we have

$$\sum_{l=1}^{m+1} \widetilde{R}_{j_l} \leqq \sum_{l=1}^{m+1} \frac{\widetilde{A}_{j_l}}{\widetilde{R}_{j_l}}\gamma \leqq \frac{1}{\widetilde{R}_{j_{m+1}}} \sum_{l=1}^{m+1} \widetilde{A}_{j_l}\gamma.$$

Using condition (A$\cdot$5), we have

$$\widetilde{R}_{j_{m+1}} \leqq \sum_{l=1}^{m+1} \widetilde{A}_{j_l}.$$

Because this satisfies condition (A$\cdot$2) we have

$$\mathcal{B}_\gamma(S_{j_{m+1}}) \leqq \mathcal{F}_\gamma(S_{j_{m+1}}).$$

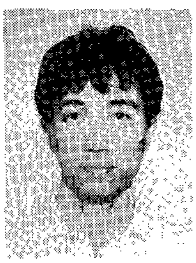
Therefore, for $0 \leqq l \leqq s-1$,

$$\mathcal{B}_\gamma(S_{j_l}) \leqq \mathcal{F}_\gamma(S_{j_l}),$$

and we conclude that for arbitrary $S \subseteqq \mathbf{N}_n$,

$$\mathcal{B}_\gamma(S_{j_0}) \leqq \mathcal{B}_\gamma(S_{j_1}) \leqq \cdots \leqq \mathcal{B}_\gamma(S)$$

holds and that for arbitrary $S_1, S_2 \subseteqq \mathbf{N}_n$, inequality (6) holds. □



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