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A Spectral-Based Model for Describing Social Polarization in Online Communities

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SUMMARY The phenomenon known as *social polarization*, in which a social group splits into two or more groups, can cause division of the society by causing the radicalization of opinions and the spread of misinformation, is particularly significant in online communities. To develop technologies to mitigate the effects of polarization in online social networks, it is necessary to understand the mechanism driving its occurrence. There are some models of social polarization in which network structure and users' opinions change, based on the quantified opinions held by the users of online social networks. However, they are based on the interaction between users connected by online social networks. Current recommendation systems offer information from unknown users who are deemed to have similar interests. We can interpret this situation as being yielded non-local effects brought on by the network system, it is not based on local interactions between users. In this paper, based on the spectral graph theory, which can describe non-local effects in online social networks mathematically, we propose a model of polarization that user behavior and network structure change while influencing each other including non-local effects. We investigate the characteristics of the proposed model. Simultaneously, we propose an index to evaluate the degree of network polarization quantitatively, which is needed for our investigations.

key words: *online social networks, polarization, Laplacian matrix, Fiedler vector*

1. Introduction

In recent years, social media, including Twitter and YouTube, have been spreading rapidly, and information exchange and dissemination on online social networks (OSNs) are being actively promoted [1]. In particular, social networking services (SNS) have become indispensable in our daily lives. By using them, we can efficiently access information matching our interests and easily communicate with other users who share common interests [2]. Although the convenience of social media has been enhanced by providing individually optimized information to each user and promoting connections among people through recommendation functions, these features have the potential for causing polarization among users [3]. In many studies, social polarization refers to the division of a social group into two groups. In this study, we consider social polarization in a more generalized way by defining it as division into two or more groups.

Many studies have addressed the occurrence of social polarization in online communications on social media. For example, [4] analyzed the Twitter discussions of several political groups and found that groups with different ideologies interact less. This tendency was particularly noticeable between groups with opposing ideologies. In addition, [5] showed that individuals with similar ideologies exchanged more information on political topics than on non-political topics. [6] and [7] pointed out that the social polarization of users may contribute to the spread of misinformation and fake news in social networks. To develop technologies to mitigate the effects of polarization, understanding the mechanism driving its occurrence is necessary.

Some of the human factors that may contribute to the occurrence of polarization include confirmation bias [8] (focusing only on information that conforms to self view) and the backfire effect [9] (reinforcing own opinions when exposed to opposing opinions). Some of the characteristics of social media include filter bubbles [10] (the preferential presentation of information of interest to each user) and echo chambers [11] (the reinforcement of user beliefs through repeated interactions within a closed community). There are many reports that these phenomena have been observed in discussions of political topics in SNS [12], [13]. By proposing a model that describes them, we believe that it is possible to establish a basis for considering universal countermeasures that can attenuate polarization in various kinds of situations.

Related work has proposed some models of polarization that posit that the quantified opinions held by users in social networks influence network structure changes and the dynamics of user opinion formation. [14] introduced a mathematical model to describe online discussions and the attendant polarization; they concluded that the repulsion of opinions among users can explain its occurrence. [15] proposed a model in which user opinions and social interactions change with the viewing of social media posts made by users. They showed that segregated communities emerge through social media mechanisms. These models describe the process by which the opinions of each user affect the network structure. However, network structure changes do not directly affect the user opinion formation. Also, though some models of polarization have been proposed in which the opinions of users and connections between users mutually interact [16], [17], each user is only directly affected by the users with whom it has some connection. Therefore, they cannot describe the non-local effects yielded by the recommendation functions of network systems. Some models have also been proposed to investigate the effect of

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recommendation functions on polarization [18], [19]. However, even in those frameworks, the opinion change of each user is not influenced by unknown users. In addition, [19] focused on the function of recommending *items*, which is often used in online shopping sites. Since the relationship between users and items is different from the relationship between users, however, its framework cannot describe the function of recommending *users* in OSNs.

In general, user interest and network structure should be related, and therefore they should interact. When the opinions and interests of each user change, the relationships among users is also expected change as a result. In addition, users can view information transmitted by unknown users in OSNs and can receive information from unknown users who are deemed to have similar interests, so they may be influenced by unknown users. These effects are non-local interactions not based on the communication through links in OSNs. Based on these considerations, we propose a model of user polarization that takes account of OSN structure. In our proposed model, we treat OSN structure in the framework of spectral graph theory. In particular, we focus on the Fiedler vector, which is a kind of eigenvector of the Laplacian matrix, for describing non-local effects. It enables us to describe the process by which user opinions and network structure change while influencing each other, as well as to model user behaviors such as following and unfollowing, that are common in social networks. Our proposed model also takes into account the unique characteristics of each user, such as leader and follower, so that it is a framework that takes a realistic view of OSNs. Our model is effective in understanding the non-local effects among users in OSNs and the impact of the interaction between opinion change and network structure change on polarization.

To evaluate the proposed model properly, we need a quantitative index to measure the degree of polarization. Many studies that propose indices of polarization focus on the division of user opinions or network structure into two groups [20], [21]. However, few indices allow for division into three or more groups. In this study, we propose an index to evaluate various types of polarization.

The rest of this paper is organized as follows. Section 2 explains spectral graph theory and the concept of the model of polarization proposed in this study. Section 3 proposes an index to quantitatively evaluate the degree of network polarization. Section 4 conducts some experiments to evaluate the model and clarify its parameters' characteristics. Finally, we state conclusions and future work in Sect. 5.

2. A Spectral-Based Model of Polarization

In this section, we describe a spectral-based model of social polarization. First, based on [22], [23], we introduce the Laplacian matrix to express network structure. Then, we describe the concept of the model that reflects user behavior and characteristics in OSNs.

2.1 Fiedler Vector and Similarity of User Opinions

Let $G(V, E)$ be an (unweighted) undirected graph representing the structure of an OSN with N nodes, where V is the set of nodes, and E is the set of undirected links. Here, nodes and links represent users and the relationships between them, respectively. When there is a link between nodes i and j, they can communicate directly. The (unweighted) adjacency matrix $\boldsymbol{A} := [A_{ij}]_{1 \le i,j \le N}$ is an $N \times N$ matrix defined as

$$A_{ij} := \begin{cases} 1, & (i,j) \in E, \\ 0, & (i,j) \notin E. \end{cases} \tag{1}$$

If node i has degree d_i, the degree matrix $\boldsymbol{D}$ is an $N \times N$ matrix defined as

$$\boldsymbol{D} := \mathrm{diag}(d_1, d_2, \ldots, d_N). \tag{2}$$

The Laplacian matrix $\boldsymbol{L}$ of $G(V, E)$ is defined as

$$\boldsymbol{L} := \boldsymbol{D} - \boldsymbol{A}. \tag{3}$$

It is known that Laplacian matrix $\boldsymbol{L}$ has a minimum eigenvalue of zero. The multiplicity of eigenvalue zero is equal to the number of connected components of $G(V, E)$.

Assume that $G(V, E)$ is a connected undirected graph. Let us sort the eigenvalues of the Laplacian matrix of $G(V, E)$ in ascending order as follows:

$$0 = \lambda_0 < \lambda_1 \le \cdots \le \lambda_{N-1}. \tag{4}$$

The smallest eigenvalue that is not zero (the second smallest eigenvalue λ_1) is called the algebraic connectivity, and it is used as a measure of how tightly connected the undirected graph is. Eigenvector $\boldsymbol{v}_1$ of $\boldsymbol{L}$ associated with λ_1 is known as the Fiedler vector [24].

Figure 1 shows an unweighted undirected graph with four densely connected subgraphs connected by some links, and Fig. 2 shows the components of the Fiedler vector of the Laplacian matrix of the graph shown in Fig. 1. From these figures, we can see that the four cluster structures of the plotted points correspond to the four subgraph structures which are characteristic of the graph. This example demonstrates

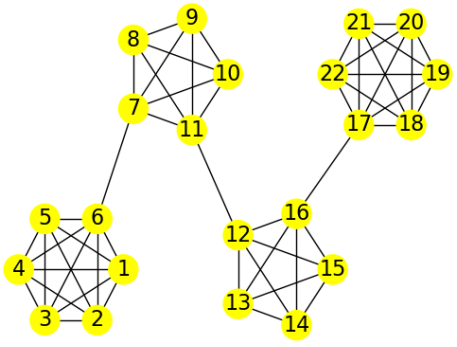


Fig. 1 Unweighted undirected graph with four densely connected subgraphs connected by some links.

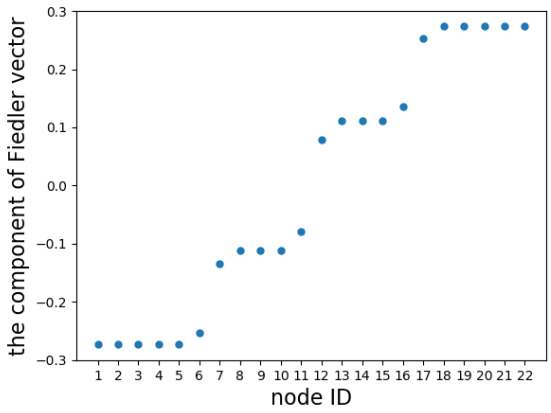


Fig. 2 Components of the Fiedler vector corresponding to Fig. 1.

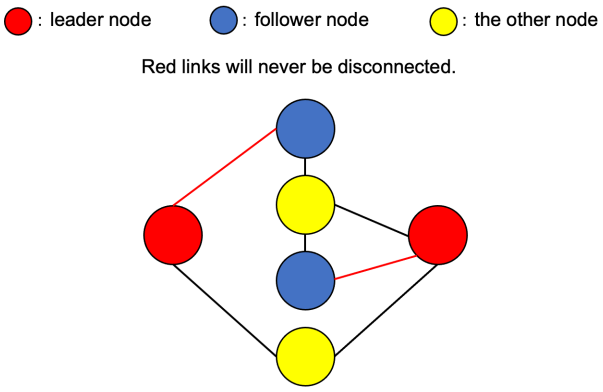


Fig. 3 Example of leader and follower nodes.

why the Fiedler vector can be taken as a useful index for understanding graph structure.

Based on the above, we can connect OSN structure and users' opinions or tastes. Components of the Fiedler vector that have similar values can be interpreted as similar user opinions or tastes. Accordingly, we propose a model of polarization that determines the opinion or taste of each node based on the value of each component of the Fiedler vector and change the network structure based on them. Here, note that the values of the components are not directly related to the strength of opinions. The fact that the values of the components of the Fiedler vector are close indicates that the corresponding users have similar opinions and tastes. This idea makes it possible to apply the model to not only topics where opinions fall into two groups, which has been the target of related work, but also topics where more diverse opinions are likely to be formed. We will describe how to determine the opinion value of each node based on each component of the Fiedler vector, along with the concept of the model of polarization.

2.2 Concept of the Model of Polarization

In social media, including SNS, each user can freely follow or unfollow other users as a response to the posts displayed on the screen. Also, we think that some users in social networks influence their surroundings by acting on their own opinions and beliefs, while others form their own opinions and build human relationships through the influence of other users. Our model of polarization reflects these user behaviors and characteristics.

To reflect the user characteristics of OSNs in our model, we introduce two types of characteristic nodes on the undirected graph representing the social network. One is the *leader node*; it represents users who act with consistent opinions and beliefs. This node has the constraint that the opinion value of the node will remain fixed at the component of the Fiedler vector in the initial state, regardless of the opinion changes caused by network structure changes. The other is the *follower node*; it represents users who form opinions and human relationships as influenced by their surroundings. This node has the constraint that it will never disconnect its link with the leader nodes. We call the node that is neither leader nor follower *the other node*. Here, we regard the opinion values of the *follower node* and *the other node* as the components of the Fiedler vector. That is, they will change with the network structure. Figure 3 shows an example of leader and follower nodes. The red links in the figure will never be disconnected even under network structure changes. Here, we assume that the characteristic of each node does not change regardless of network structure changes.

Next, we explain the procedures for disconnecting and connecting links to describe the changes in the relationships between users. At each time step, we select a node at random and find its adjacent node whose opinion value is the most distant from that of the selected node. Then, we disconnect the link between them only if both of the following two conditions are satisfied:

- The link does not connect a leader node to a follower node.
- The graph remains connected if we disconnect the link.

The process of disconnecting links recreates the fade out of relationships between users with different opinions and tastes.

Then, if we disconnected the link, we select one node that is not adjacent to the first selected node and connect a new link between them. Here, we provide two ways of selecting the node:

(i) With probability ϕ, we randomly select one node from the set of nodes that are not adjacent to the first selected node.
(ii) With probability $1 - \phi$, we select one node that is not adjacent to the first selected node and that has the closest opinion value to it.

Since (i) is a link connection process that does not depend on the opinion value of each node, we can assume that it corresponds to a coincidental encounter in social networks. On the other hand, (ii) expresses the process of connecting people with similar opinions and tastes, so we can assume that it corresponds to non-local effects such as the encounters created by the recommendation functions in social media.

In the environment of OSNs, random link connections as in (i) are rare. Therefore, we believe that setting ϕ to a small value can realistically represent the process of building relationships between users in OSNs.

According to the rules described above, OSN structure and the opinion value of each node will change while influencing each other by repeating the process of link disconnection and connection. This model does not change the total number of links, but the degree of each node can change.

3. Index of Social Polarization

In Sect. 2, we proposed a model of polarization for changing the network structure based on the opinion value of each node. However, to determine whether the network after a structure change is polarized or not, a quantitative measure is necessary. Clustering coefficients are attractive as indices to evaluate the strength of polarization. This is because sparse and dense parts are visible in a polarized network structure, and the corresponding clustering coefficient is expected to be high. However, it is difficult to distinguish between dense networks (e.g., complete graphs) from polarized networks using clustering coefficients because those graphs also have high values. Moreover, they cannot determine how many groups exist in the network. For these reasons, we believe that clustering coefficients cannot be used to quantify polarization. Therefore, in this section, we propose an index of polarization that describes both the number of polarized groups and the strength of the polarization effects in the network based on the distribution of the opinion value of each node. The procedure to derive the index includes two steps: determination of the number of polarized groups, and evaluation of the strength of the polarization effects.

We start by deleting the nodes with degree 1 from the network. This operation corresponds to deleting terminal nodes in network analysis when extracting the network structure of an SNS. This process prevents us from being unable to capture the rough characteristics of the network structure due to the influence of small degree nodes.

Next, we normalize the opinion value of each node. This process enables us to handle the component distribution of the Fiedler vector, which greatly depends on the number of nodes in the network and its structure, in a unified manner. Let o_i be the opinion value of node i, $o_{\min}$ be the minimum opinion value among all nodes, and $o_{\max}$ be the maximum value. We define the normalized opinion value o'_i of node i as follows:

$$o'_i = \frac{2\,(o_i - o_{\min})}{o_{\max} - o_{\min}} - 1. \tag{5}$$

The minimum and maximum values of the normalized opinion value are -1 and $+1$, respectively.

Next, we evaluate the number of polarized groups in the network and the number of nodes in each polarized group based on the distribution of normalized opinion values. We divide the range $[-1, +1]$ of normalized opinion values evenly into n intervals, and obtain the frequency distribution by counting up the number of opinion values in each interval. If the frequency in an interval is greater than or equal to a threshold θ, then we consider the nodes with opinion values in the interval to be one group. That is, threshold θ represents the minimum number of nodes that can be considered as a group. In addition, threshold θ restricts the maximum number of polarized clusters. Here, we introduce some conditions that should be satisfied between the scale of the network and the number of polarized groups contained in it:

- As the scale of the network grows, the number of polarized groups that can be formed increases.
- A group with a very small scale relative to the scale of the entire network is not considered a polarized group.

By considering the above two conditions, we define threshold θ as

$$\theta = \left\lfloor \sqrt{N} \right\rfloor, \tag{6}$$

where N is the number of nodes in the network and $\lfloor x \rfloor$ is the largest integer less than or equal to x. Based on the above, let m_n be the number of polarized groups in the network and $\boldsymbol{g}_n$ be the m_n-dimensional vector with the number of nodes in each polarized group as each component, where n is the number of intervals. Note that the range constraint on n, described below, guarantees $m_n \geq 1$.

As an example, we obtain m_n and $\boldsymbol{g}_n$ from the components of the Fiedler vector shown in Fig. 2, that is, the opinion value of each node. Figure 4 shows the opinion values normalized to the range $[-1, +1]$ using (5). Table 1 shows the frequency distribution table when the number of intervals, n, is four. Threshold $\theta = 4$, since the number of nodes $N = 22$. From Table 1, since the frequencies of all intervals are greater than or equal to θ, we obtain $m_4 = 4$ and $\boldsymbol{g}_4 = (6, 5, 5, 6)$.

Using m_n and $\boldsymbol{g}_n$, we define index $P(n)$ for the degree of polarization of the network structure as follows:

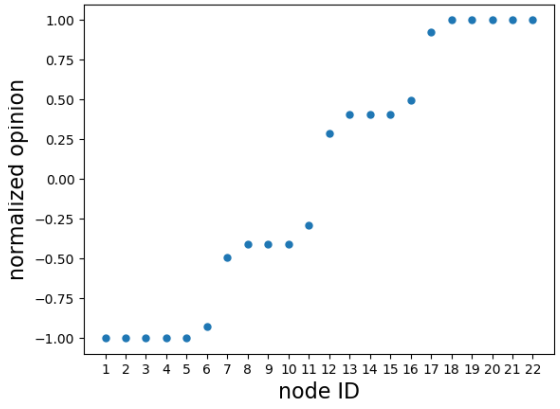


Fig. 4 Normalized opinion values.

Table 1 Frequency distribution table ($n = 4$).

Opinion value	Frequency
−1.0∼−0.5	6
−0.5∼0.0	5
0.0∼0.5	5
0.5∼1.0	6

Table 2 Example of calculating each index.

n	m_n	$\boldsymbol{g}_n$	$P_1(n)$	$P_2(n)$	$P(n)$
3	2	(10, 10)	0.909	2.00	0.525
4	4	(6, 5, 5, 6)	1.00	1.00	0.707
5	4	(6, 5, 5, 6)	1.00	1.00	0.707

$$P(n) := \begin{cases} 0, & m_n = 1, \\ \dfrac{P_1(n)}{\sqrt{P_2(n)+1}}, & m_n \geq 2. \end{cases} \tag{7}$$

Here, $P_1(n)$ and $P_2(n)$ in (7) are defined as

$$P_1(n) := \frac{1}{N} \sum_{i=1}^{m_n} g_n(i), \tag{8}$$

$$P_2(n) := \sum_{i=1}^{m_n} \left(\frac{N}{m_n} - g_n(i) \right)^2, \tag{9}$$

where $g_n(i)$ is the i-th component of vector $\boldsymbol{g}_n$. $P_1(n)$ is an index that indicates how many nodes belong to some polarized group out of all the nodes in the network. The larger the value of $P_1(n)$ is, the greater the degree of polarization is. $P_2(n)$ is an index that indicates how unbalanced the number of nodes in each group is. The smaller the value of $P_2(n)$ is, the greater the degree of polarization is. $P(n)$ is determined by $P_1(n)$ and $P_2(n)$, and the range of its value is $[0, 1]$, where the larger the value is, the more strongly polarized the network structure is. In addition, since $P_1(n)$ and $\sqrt{P_2(n)+1}$ have the same dimension, index $P(n)$ is a dimension-less measure. The reason why $P(n) = 0$ when $m_n = 1$ is because we define social polarization as division into two or more groups, as described in Sect. 1.

Finally, we calculate $P(n)$ for each integer n satisfying $3 \leq n \leq [N/\theta]$ and define the index P^* of polarization as follows:

$$P^* := \max_n P(n). \tag{10}$$

Based on the above, we quantitatively determine the degree of polarization from P^* and the number of polarized groups on the OSN from m_n used to calculate P^*.

Table 2 shows examples of calculating each index from the distribution of opinion values shown in Fig. 4 in the range $3 \leq n \leq [N/\theta] = 5$. Note that $P_1(n)$, $P_2(n)$, $P(n)$ values are written to three significant digits. From Table 2 and Eq. (10), the degree of polarization of the network structure shown in Fig. 1 is calculated as $P^* = 0.707$. Considering this result together with the fact that the number of polarized groups when $P(n)$ is maximum is $m_n = 4$, we believe that the values of P^* and m_n appropriately reflect the characteristics of the network structure shown in Fig. 1.

4. Basic Characteristics of the Proposed Social Polarization Model

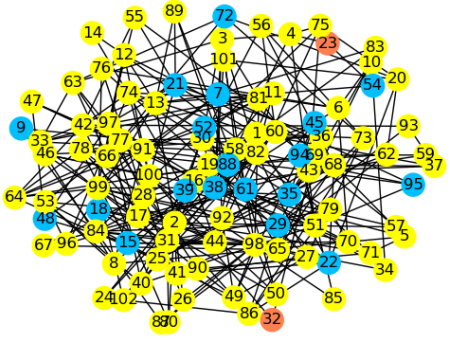
In this section, we examine through simulations how the model parameters and the characteristics of nodes introduced in the model described in Sect. 2 affect the social polarization of the network structure.

The experiments use unweighted undirected graphs with the number of nodes $N = 102$ generated by the Erdös-Rényi model (ER model) [25] and the Barabási-Albert model (BA model) [26] as network models. Here, the link generation probability in the ER model is set to 0.05, and the number of nodes in the initial state in the BA model is set to three. Figures 5 and 6 illustrate the corresponding network structures. The red nodes represent leader nodes, the blue nodes represent follower nodes, and the yellow nodes represent the other nodes. The number of leader and follower nodes in Fig. 5 is 2 and 19, respectively, and in Fig. 6 it is 2 and 44, respectively. In the upper panels of Figs. 5 and 6, leaders are the nodes with the largest or smallest opinion values, and followers are selected randomly. On the other hand, in the lower panels of Figs. 5 and 6, leaders are the nodes with the first or second highest degree, and followers are the nodes adjacent to the leader. We apply the model to each network and calculate the indices P^* and m_n from the network after structure changes. As an experimental condition, the simulation time for the model, t, was set to 3000. Also, from (6), the threshold used to calculate the indices was set at $\theta = 10$.

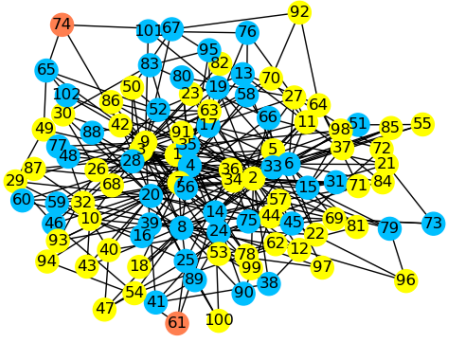
Figures 7 and 8 show index, P^*, of polarization and the number, m_n, of polarized groups for various ϕ values, using the networks shown in the upper and lower panels of Fig. 5 as the initial conditions, respectively. We conducted the simulation 100 times for each value of ϕ by changing the seed used in random node selection at each time step, so P^* and m_n in the figures are averages. Note that the error bars represent the 95% confidence interval.

Focusing on the averages of P^* in Figs. 7 and 8, we can see that P^* takes larger values as parameter ϕ decreases. As mentioned in Sect. 2, a smaller value of ϕ means that users with similar opinions are more likely to connect as a result of the network structure change. Also, we design the model of polarization so that the links between nodes with different opinions and interests are disconnected. Therefore, these results indicate that in the random network generated by the ER model, polarization is more likely to occur due to these two factors, regardless of how the leader and follower nodes are defined.

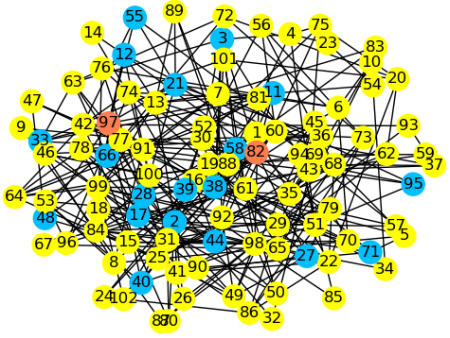
Next, focusing on the averages of m_n in Figs. 7 and 8, we can see that decreasing ϕ increases the value of m_n. This indicates that the tendency for users with similar opinions to connect contributes to forming many polarized groups in the network. Here, for $\phi = 0$ to 0.3, the averages of m_n are greater than two, even though the number of leader nodes is two. It means that groups without leader nodes are likely to appear in the network. We believe that the reason is the



Leader nodes have the largest or smallest opinion values;
follower nodes are selected randomly

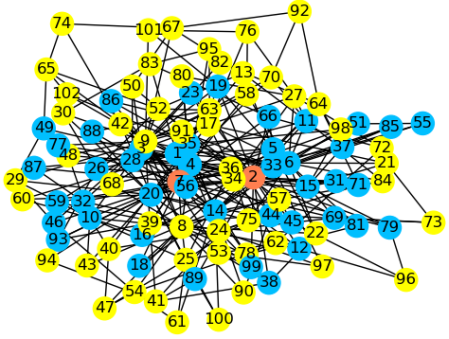


Leader nodes have the largest or smallest opinion values;
follower nodes are selected randomly



Leader nodes have the first or second highest degree;
follower nodes are adjacent to the leader

Fig. 5 Unweighted undirected graph generated by the ER model (N = 102).



Leader nodes have the first or second highest degree;
follower nodes are adjacent to the leader

Fig. 6 Unweighted undirected graph generated by the BA model (N = 102).

degree of the leader nodes in their initial state. When the leaders are the nodes with the largest or smallest opinion values, the smaller ϕ is, the more difficult it is for them to acquire more links because their opinion values are far from those of the other nodes. Also, when the leaders are the nodes with large degree, their degree is at most one-tenth of the total number of nodes in the network. Even if a group including the leader is formed, its scale is not expected to be very large. Therefore, there are many nodes that do not form a group with a leader, and the number of polarized groups without a leader will increase.

To further understand the influence of the characteristics of the nodes introduced in our model, we also simulated the situation in which leaders change their opinions (Appendix). The simulation results indicate that whether the leaders change their opinions or not is not an important factor in polarization. Thus, we reiterate our assumption that the leader holds a consistent opinion. In the experiments using the network generated by the BA model, we do so under the condition that leaders keep their opinions unchanged.

Figures 9 and 10 show index P^* of polarization and the number m_n of polarized groups for various ϕ values, using the networks shown in the upper and lower panels of Fig. 6 as initial conditions, respectively. The number of simulations and the meaning of the error bars are the same as those mentioned above.

Figure 9 indicates that decreasing ϕ increases P^* and m_n. It is similar to the results of the network generated by the ER model. Therefore, under this initial state, we can confirm that the tendency to connect users with similar opinions also contributes to polarization in the scale-free network generated by the BA model.

However, focusing on the averages of P^* in Fig. 10, we can see that the value of P^* is small regardless of the value of ϕ. This result is very different from the characteristic mentioned above. To discuss the reason for this, let us consider the networks in the initial state. In the network generated by the BA model, nodes with a large degree tend to connect, so the opinion values of the leader nodes are close to each other. In addition, the followers determined as the adjacent nodes of the leaders have the constraint of keeping their links with the leaders, so their opinion values tend to approach that of the leader. Therefore, we believe that index P^* of polarization is small due to the formation of a large-scale group that includes two leader nodes and many follower nodes.

Focusing on the averages of m_n in Fig. 10, we can see

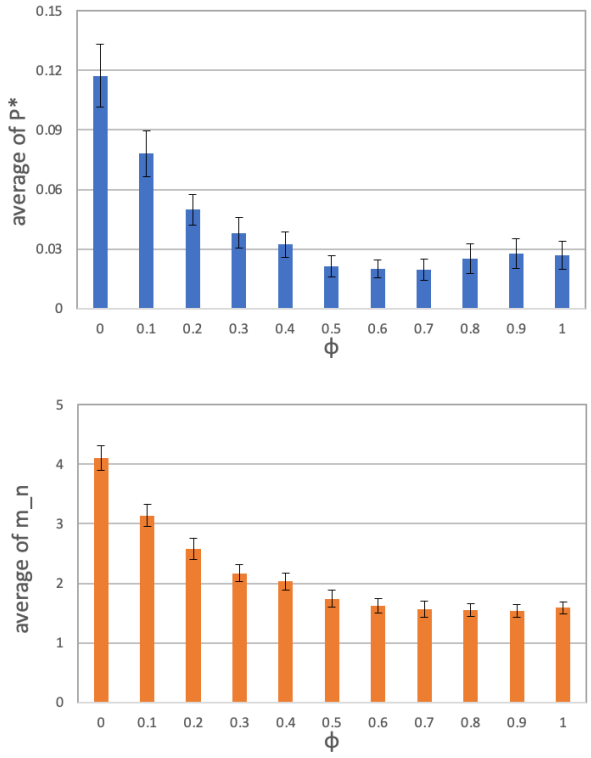


Fig. 7 Average of P^* and m_n for parameter ϕ using the upper panel of Fig. 5 as the initial network.

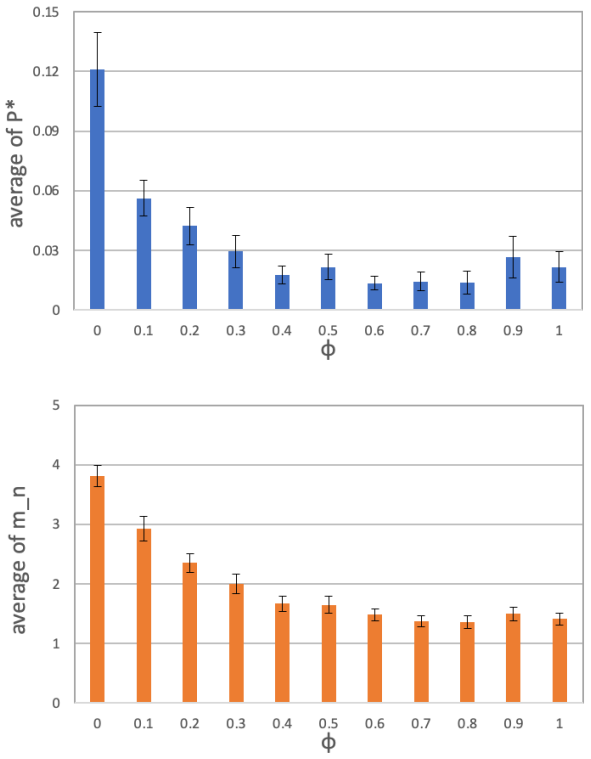


Fig. 8 Average of P^* and m_n for parameter ϕ using the lower panel of Fig. 5 as the initial network.

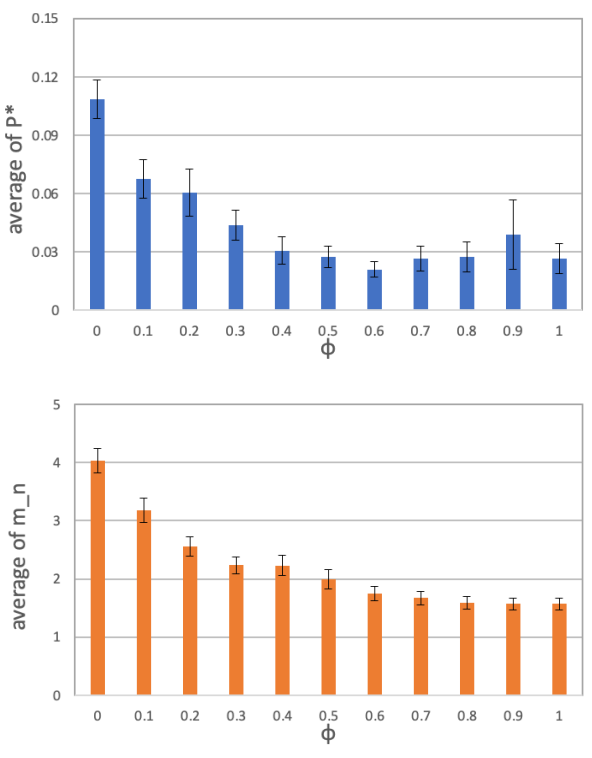


Fig. 9 Average of P^* and m_n for parameter ϕ using the upper panel of Fig. 6 as the initial network.

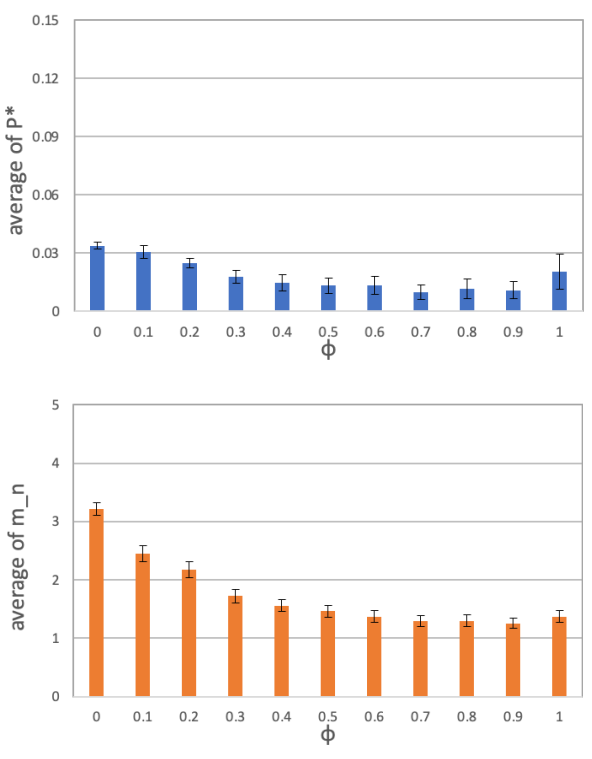
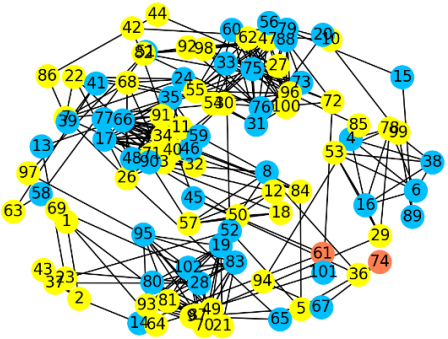


Fig. 10 Average of P^* and m_n for parameter ϕ using the lower panel of Fig. 6 as the initial network.



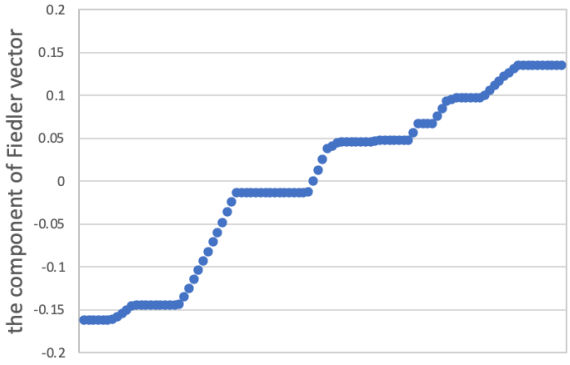
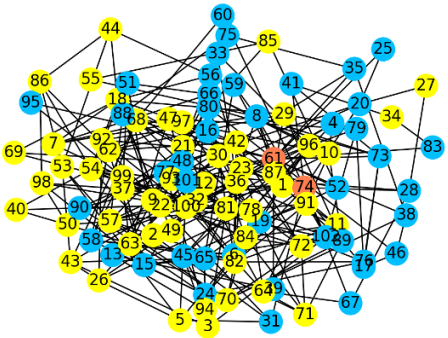


Fig. 11 Network and the components of Fiedler vector in ascending order ($t = 3000$, $\phi = 0$, $P^* = 0.176$).



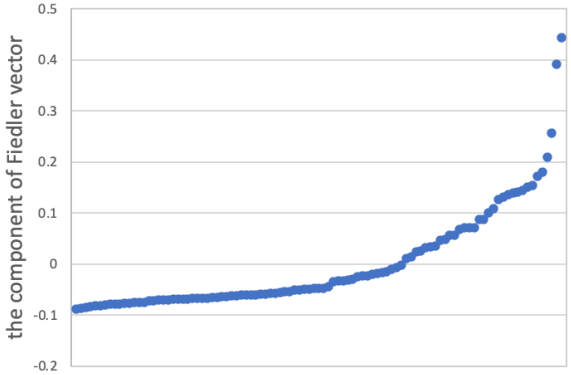


Fig. 12 Network and the components of Fiedler vector in ascending order ($t = 3000$, $\phi = 1$, $P^* = 0.025$).

that decreasing ϕ increases m_n. However, comparing the averages of m_n shown in Figs. 7–10, we find that those of Fig. 10 are the smallest overall among those of the others. We think that this is due to the existence of the large-scale group discussed in the previous paragraph. The formation of a large-scale group means that the number of nodes not included in it tends to be small. Therefore, it is hard for those nodes to form a new group.

Here, we describe further experimental results, focusing on the scale-free network generated by the BA model. To facilitate understanding of the relation among index P^* of polarization, the network topology, and the distribution of opinion values, we show some examples of them after structure change.

Figure 11 shows the network and the components of the Fiedler vector in ascending order at $t = 3000$ with $P^* = 0.176$. For the experimental conditions, we used the network shown in the upper panel of Fig. 6 as the initial condition with parameter $\phi = 0$. These figures indicate that there are multiple community structures in the network and that the components of the Fiedler vector distribute in a staircase pattern with several peaks. This result shows that a polarized structure appears in the network even if P^* is around 0.15, although it is far from the *complete* polarization represented by $P^* = 1$. It is also notable that leader nodes do not belong to any community in the network. The reason may be that it is hard for the leaders to acquire links because random link connections cannot occur under this experimental condition. Nevertheless, such a polarized situation occurs regardless of leaders because each node connects with nodes that have similar opinions and blocks nodes that have different opinions.

Figure 12 shows the network and the components of the Fiedler vector in ascending order at $t = 3000$ with $P^* = 0.025$. For the experimental conditions, we used the network shown in the upper panel of Fig. 6 as the initial condition with parameter $\phi = 1$. Under this condition, there is no community structure in the network, and multiple peaks do not appear in the distribution of the components of the Fiedler vector. In this case, even if it is determined by the calculation of the index that there are several groups in the network, the number of nodes among them is highly unbalanced. Therefore, the value of P^* is smaller than that of Fig. 11.

Finally, to understand how polarization strength changes with time, we investigate the time evolution of polarization index P^* and the number of polarized groups, m_n. Figure 13 shows the simple moving averages of the time evolution of P^* and m_n for parameter ϕ using the upper panel of Fig. 6 as the initial condition. From these figures, we can see that P^* and m_n decrease regardless of the value of ϕ at the beginning, but then increase as the value of ϕ decreases. That is, we can assume that the process of polarization begins with the formation of a large-scale group, which then gradually divides into multiple groups. Also, under $\phi = 0$,

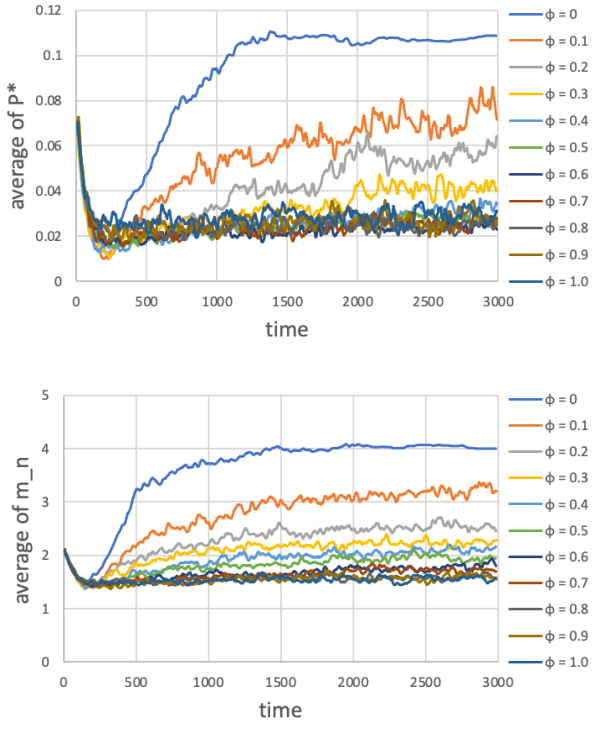


Fig. 13 Time evolution of P^* and m_n for parameter ϕ using the upper panel of Fig. 6 as the initial condition (simple moving average of interval width 19).

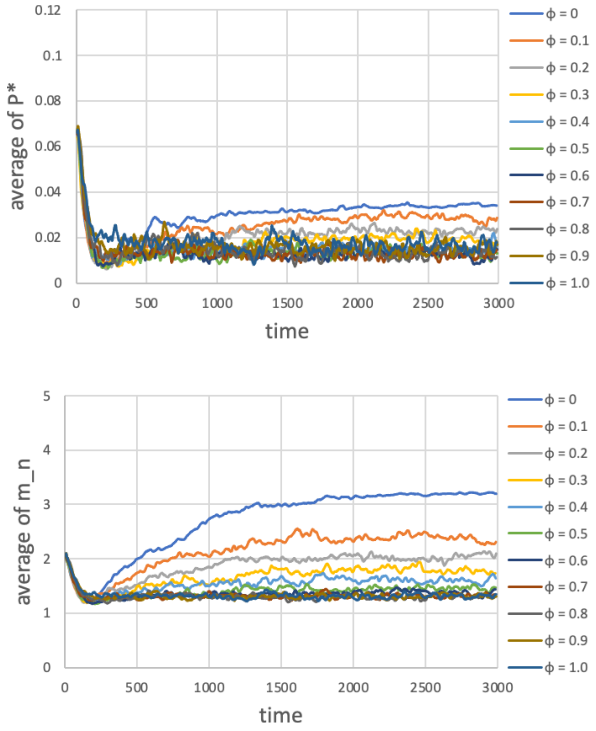


Fig. 14 Time evolution of P^* and m_n for parameter ϕ using the lower panel of Fig. 6 as the initial condition (simple moving average of interval width 19).

where there is no randomness in the link connection, the values of P^* and m_n are stable in the latter half of the simulation time. On the other hand, under $\phi = 0.1$ to 0.3, where random link connections occasionally occur, they seem to have increasing tendencies even at $t = 3000$. These results indicate that random link connections may have the effect of slowing or stopping the progression of polarization.

Figure 14 shows the simple moving averages of the time evolution of P^* and m_n for parameter ϕ using the lower panel of Fig. 6 as the initial condition. Focusing on the time evolution of P^*, it hardly increases after decreasing regardless of the value of ϕ. The reason seems that the constraint that the link between a leader and a follower is never disconnected prevents the division of the large-scale group. However, since the other nodes do not have such a constraint, they can form small groups. It may be related to the result that m_n increases with time when ϕ is small.

5. Conclusion

In this paper, we use the components of the Fiedler vector in spectral graph theory to propose a model of social polarization in which user behavior and network structure change while influencing each other. We also proposed an index that can evaluate social polarization quantitatively. Our model reflects the non-local effects of user behavior, and the characteristics of users on OSNs. Simulations revealed that social polarization is more likely to occur due to the synergistic effect of connecting users who have similar opinions with attenuation of relationships between users who have different opinions. We also found that when the hubs in the scale-free network are leaders and the users around them are followers, a large-scale group including them can readily form as the network structure changes. Meanwhile, we found that increasing the number of interactions that are independent of user opinion values mitigated the polarization regardless of the initial network structure and how leaders and followers were determined, which is seen as an important finding for developing countermeasures to polarization.

In future work, we will examine the impact of the initial network structure on polarization in more detail. We will also consider extending the applicability of the model so that it can handle weighted undirected graphs and directed graphs.

Acknowledgments

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Appendix: Experimental Results in the Case Where the Leaders' Opinions are Variable

In the main text, we assumed that leaders have consistent opinions and that the opinion values of the leader nodes remain unchanged regardless of network structure changes. However, in some cases, leaders may pander to the opinions of their followers and change their own opinions. In this section, we show some simulation results when the opinion values of all nodes, including leader nodes, are regarded as components of the Fiedler vector.

Figure A· 1 shows the results when the upper panel of Fig. 5 is the initial condition and the leaders' opinions are variable. Compared to the results when the leader's opinion is invariant (Fig. 7), there are slight differences in the averages of index P^* of polarization and the number m_n of polarized groups, but the tendency that they increase as ϕ decreases remains. These results imply that whether networks and opinions become polarized or not is mostly independent of whether leaders change their opinions or not.

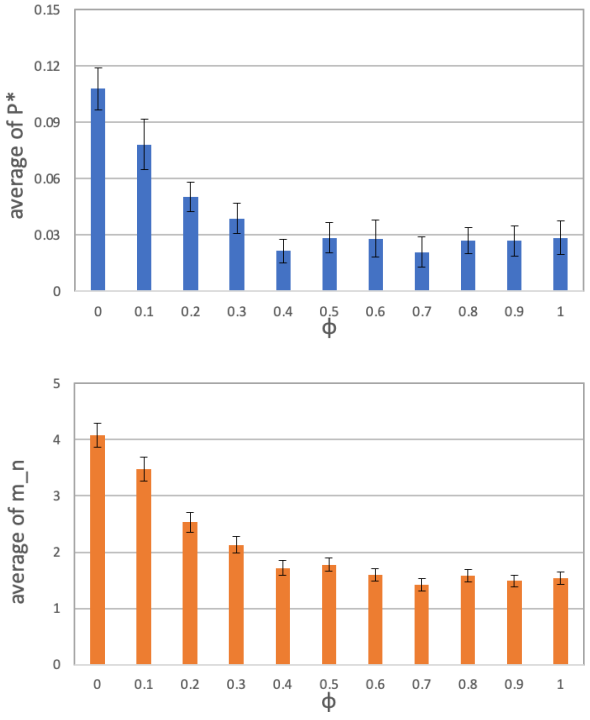

Fig. A· 1 Average of P^* and m_n for parameter ϕ in the case where the leaders' opinions are variable (initial network: the upper panel of Fig. 5).

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