Achieving Power-law Placement in Wireless Sensor Networks

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Abstract

One of the most important issues in sensor networks is to develop the technology for improved fault tolerance. From this viewpoint, the placement of sensor nodes is critical. To date, we have proposed stochastic node placements whose degrees of nodes follow a power-law ("power-law placement"). To demonstrate the effectiveness of power-law placement, we have to show that this placement has high fault tolerance and can be achieved with a reasonable degree of complexity and accuracy. In the first step of our research, we have shown that power-law placement with well-tuned parameters is more robust against failure than general stochastic node placement. In the second step to prove the effectiveness of power-law placement, we investigate ways of achieving power-law placement in this paper.

1. Introduction

Since networks of sensors with communication capabilities would enable information to be gathered over wide areas, interest in sensor networks is currently attracting a great deal of interest. Sensor nodes are prone to failure and have limited power capacity, so the creation of technology to increase fault tolerances is also a major issue in sensor networks. Although sophisticated routing and a MAC layer protocol are expected to improve fault tolerances, placement of sensor nodes also strongly affects this tolerance in a sensor network. However, little research into placement has been done. In addition, all research to date has been based on deterministic node placement [1, 2, 3], which requires each sensor node be placed at predetermined coordinates. This has following shortcomings, high cost of placement and unknown effects of errors in sensor positions.The alternative approach is stochastic node placement, where the sensor-positions are determined by a probability density function. This approximate positioning that satisfies the density function should lower the cost of placement. The concept of stochastic node placement is shown in Fig. 1. Motivated by these considerations, we investigated stochastic node placements that had a high degree of fault tolerance and were easily achieved. We have proposed a new form of stochastic node placement, i.e., power-law placement such that the degree of the nodes followed a power-law [6]. In the first step to demonstrate the effectiveness of power-law placement, we evaluated the relative fault tolerance of power-law placement and the two most typical forms of stochastic placement. Through evaluations, we found that power-law placement with well-tuned control parameters had higher tolerance against failure of nodes. In the second step to demonstrate the effectiveness of power-law placement, we also should show how power-law placement can be achieved with reasonable complexity and accuracy. Therefore, this paper examines ways of attaining power-law placement.

The rest of this paper is organized as follows. Sect. 2 covers the communications model and simulation scenarios for evaluating fault tolerance. Sect. 3 introduces our previous research on the fault tolerance of sensor networks. Sect. 4 proposes ways of achieving power-law placement. Our proposal to attain power-law placement is based on the consideration that any stochastic node placement can be achieved by scattering sensor nodes from the air. After the theoretical foundation for this consideration is described, we propose two ways of implementation. Sect. 5 covers evaluations on each implementation. Finally, Sect. 6 presents a summary.

2. Background

2.1. Communications Model

We will explain how a sensor network senses and transmits data by referring to Fig. 2. This is a situation where the precise location of the target is unknown but is known to be in a certain region. Here, we have assumed that the target is within region D.

Each sensor node has a specific sensing range for a target and sends sensed information to the base station. A sensor node can transmit data to or receive data from other sensor nodes within its radio-transmission range.

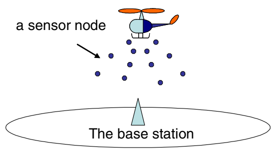

Figure 1. Stochastic node placement

A route to the base station is selected by a minimum-hop strategy. A sensor node consumes battery energy in transmitting and receiving bits. When a sensor node uses up its battery energy, all functions of the node stop. If a relay node on the current route breaks down, an alternative route is selected for the remaining information.

Note that we refer to a target being sensed as successful when one or more sensor nodes is within sensing range of the target, and at least one of these nodes has a route to the base station.

2.2. Evaluating fault tolerance

We will describe a performance metric and simulation scenarios to evaluate fault tolerance.

We adopted the virtual sensing-success ratio as a performance metric. This refers to the probability that a given target uniformly appeared in region D will be successfully sensed. Using this metric, we evaluated the fault tolerance (i.e., sensing-success ratio) at arbitrary points in time.

We considered scenarios where the number of live sensor nodes gradually decreases because of random errors or battery exhaustion.

In evaluating tolerance against random failures, we evaluated the virtual sensing-success ratio for various values of r_b, the proportion of broken nodes. Broken nodes were randomly selected for each value of r_b.

In evaluating tolerance against battery exhaustion, we considered a situation where the number of nodes that have used up their battery energy increases with the number of targets that were uniformly appeared within region D. We evaluated the relationship between virtual sensing-success ratio and the number of targets appeared. In the evaluation, we set up the target-sensing period, T, to follow an exponential distribution with an average of 72 min. During target-sensing period, the transmission interval was 18 min and the data volume was 150 kbytes. A new target was appeared T_I after the end of the target-sensing period for the previous target. We set T_I to follow an exponential distribution with an average of 250 sec.

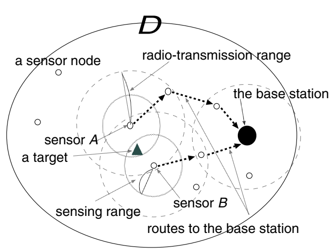

Figure 2. Sensing and transmitting data

The number of sensor nodes, N, was 250. We set the sensing range to 60 m and the radio-transmission range to 100 m. Energy consumption was 3.3e-07 J/bit for transmission and 1.9e-07 J/bit for reception [4]. The initial energy of each sensor node was 20 J.

3. Our previous results

We have proposed a new form of stochastic node placement, "power-law placement" whose degrees of nodes followed a power-law [6]. We compared the fault tolerance of sensor networks configured by power-law placement with those configured by two typical placements. In this section, we briefly introduce our results.

3.1. Stochastic node placement

We describe three types of stochastic placement we have evaluated in [6]. Two are the most typical types, i.e. simple diffusion and constant placement, while the third is our proposal, power-law placement.

In stochastic node placement, sensor-positions $\boldsymbol{x} \in \mathbb{R}^2$ are defined by a probability density function (p.d.f.), $f(\boldsymbol{x})$. [1] We assumed that all sensor nodes would be placed in region D, which is a circle with radius R. Under this assumption, $\int_D f(\boldsymbol{x}) d\boldsymbol{x} = 1$ is required. We located the base station at the center of this circle, and treated this as the origin.

Simple diffusion The simplest way to distribute sensor nodes is to scatter them from the air above the base station. This is called simple diffusion. Suppose that the weight and shape of the sensor nodes are such that air drag has a strong effect.If air current is sufficiently weak, placement of the sensor nodes will be randomized, so that it follows a diffusion equation. Since the solution to a diffusion equation on a two-dimensional boundary is a two-dimensional normal distribution, the p.d.f. of the sensor-positions is

$$f(\boldsymbol{x}) = \frac{C}{2\pi\sigma^2} \exp(-\frac{\|\boldsymbol{x}\|}{2\sigma^2}), \tag{1a}$$

$$C = \frac{1}{1 - \exp(-\frac{R^2}{2\sigma^2})}, \; 0 \le \|\boldsymbol{x}\| \le R. \tag{1b}$$

In Eq. (1), σ^2 is the variance in distribution.

Constant placement In much of the work, placement to achieve a uniform density has been assumed. We call this constant placement.

[1] The probability of a sensor being within region $A = \{x_{11} \le X_1 \le x_{12},\ x_{21} \le X_2 \le x_{22}\}$ can be written in terms of the p.d.f. as follows: $P\{x_{11} \le X_1 \le x_{12},\ x_{21} \le X_2 \le x_{22}\} = \int_{x_{21}}^{x_{22}} \int_{x_{11}}^{x_{12}} f(x_1, x_2) dx_1 dx_2$.

Power-law placement (our proposal) The p.d.f. of sensor-positions at polar-coordinates $f_p(r,\ \theta)$ is

$$f_p(r,\theta) = \frac{\alpha+1}{2\pi R}\left(\frac{r}{R}\right)^{\alpha},$$
$$0 \le r \le R,\ 0 \le \theta < 2\pi, -1 < \alpha < 1. \quad (2)$$

When the radius of region D is much larger than the radio-transmission range, the asymptotic behavior of the degree of the nodes $g(x)$ is proportional to $x^{\frac{2}{\alpha-1}}$. This means the degree of the nodes follows a power law.

The characteristics of various placements are indicated by the p.d.f. plots in Fig. 3. Region D is a circle with a radius of 500 m, centered on the base station. Variance for simple diffusion has been set so that 99% of the sensors are within region D. In the other placements, all sensor nodes are within region D. The power-law placements have greatest density near the base station, and this rapidly decreases with distance and then remains almost constant. As α increases, density decreases near the base station and increases around the border of region D.

3.2. Fault tolerance of sensor networks configured by stochastic node placement

We will summarize fault tolerance of sensor networks configured by each stochastic node placement.

To keep the virtual sensing-success ratio high when failure of nodes occurs, we must increase the probability of there being some sensor nodes within the sensing range of each target, and that of at least one of these nodes having routes to the base station. When there are few broken nodes, the former probability affects the virtual sensing-success ratio more, and when there are more broken nodes, the latter one does. Fault tolerance becomes higher by using placement where both probabilities are well balanced when failure of nodes occurs.

The former probability is high when the density of sensor nodes is uniform. The latter is high when sensor nodes are more dense near the base station, since sensor nodes near the base station have a higher probability of being used as a relay node. Since sensor nodes in simple diffusion are less dense near the border of region D (see Fig. 3), the former probability is low. Since sensor nodes in constant placement are not so dense near the base station, the latter probability is low. As a result, both have poor fault tolerance. However, power-law placement can increase fault tolerance with appropriately selected control parameters (i.e., $-0.1 \le \alpha \le 0.0$).

4. Power-law placement achieved with simple diffusion

We will propose ways of achieving power-law placement with $\alpha = 0$ using superposition of simple diffusion (remember that $\alpha = 0$ is one of the best parameters). We use simple diffusion since simple diffusion is one of the easiest ways to place sensor nodes (i.e., only scattering sensor nodes from the air achieves simple diffusion).

4.1. Theoretical basis for our proposal

In simple diffusion, p.d.f. of the sensor-positions is a two-dimensional normal distribution. This is known as a radial basis function. Daubechies [5] showed that any functions $F(\boldsymbol{x})$ can be expressed in terms of one or more radial basis functions as follows:

$$F(\boldsymbol{x}) = \int p(\boldsymbol{c},\sigma)h(\boldsymbol{x};\boldsymbol{c},\sigma)d\boldsymbol{c}d\sigma, \quad (3a)$$
$$h(\boldsymbol{x};\boldsymbol{c},\sigma) = \exp(-\frac{\|\boldsymbol{x}-\boldsymbol{c}\|}{2\sigma^2}). \quad (3b)$$

In Eq. (3), $p(\boldsymbol{c},\sigma)$, $\boldsymbol{c}$, and σ^2 are the weights, center and variance for each normal distribution. This expression means infinite superposition of simple diffusion can lead to any form of stochastic sensor-placement. Since this infinite superposition is hard to implement, we replace the integral in Eq. (3) with the finite superposition of $h(\boldsymbol{x};\boldsymbol{c}_i,\sigma_i)$ as follows:

$$F(\boldsymbol{x}) \approx \textstyle\sum_{j=1}^{M} p_j(\boldsymbol{c}_j,\sigma_j)h(\boldsymbol{x};\boldsymbol{c}_j,\sigma_j), \quad (4a)$$
$$h(\boldsymbol{x};\boldsymbol{c}_j,\sigma_j) = \exp(-\frac{\|\boldsymbol{x}-\boldsymbol{c}_j\|}{2\sigma_j^2}). \quad (4b)$$

In Eq. (4), $p_j(\boldsymbol{c}_j,\sigma_j)$, $\boldsymbol{c}_j$, and σ_j^2 are the weights, center, and variance for each normal distribution. M means the number of superpositions.

4.2. Parameters settings for simple diffusion

Achieving power-law placement is equivalent to determining parameters in Eq. (4). For implementation simplicity, it is desirable that the amount of scattering is small and that a regulation on the place of the centers is imposed. According to this consideration, we propose two fundamental ways of determining the parameters in Eq. (4). Note that other ways such as combination of both ways are easily achieved by applying these two.

Simple diffusion with fixed centers In this way, centers of simple diffusion are placed in the same position as the base station (we call this "simple diffusion with fixed centers"). This corresponds to scattering M types of sensor nodes with different form or weights (for different σ_i) simultaneously in the air above the base station. This is the simplest way to implement Eq. (4), since only once scattering achieves this. Simple diffusion with fixed centers can be used when the area where the sensor nodes are placed is relatively small. This is because there is an upper bound on the area where the sensors can reach using simple diffusion.

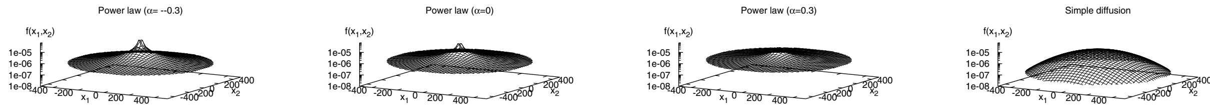

Figure 3. P.d.f. plots for various placements

<u>**Simple diffusion with concentric centers**</u> To cope with a large region D, we propose to place centers of simple diffusion on concentric circles. (we call this "simple diffusion with concentric centers"). This corresponds to scattering sensor nodes from helicopters flying on concentric circles. An example of center's positions is shown in Fig. 4.

Next, we will describe each way in detail.

4.3. Simple diffusion with fixed centers

In this implementation, Eq. (4) is rewritten as follows:

$$F(x) \approx \sum_{j=1}^{M} \frac{p_j}{2\pi\sigma_j^2} h(\boldsymbol{x}; \boldsymbol{0}, \sigma_j). \tag{5}$$

We determined the parameters in Eq. (5) through following strategies.

Since smaller value of M is better for implementation simplicity, we minimize the value of M.

Under the given value of M, we formulate the optimization problem to determine the parameters (σ_j, p_j). In this problem, we minimize sum of differences between the approximated p.d.f. and that of power-law placementon $\boldsymbol{x}_i$ that are uniformly distributed over region D. We also add the constraint $\sum_{j=1}^{M} p_j < P$, $(P \geq 1)$ to set the upper bound for the number of sensor nodes.

To evaluate the validity of $F(\boldsymbol{x})$, we compare the fault tolerance of this approximated placement with that of power-law placement.In evaluation, we use a performance metric (i.e., the virtual sensing-success ratio) and simulation scenarios described in Sect. 2.2. We consider $F(\boldsymbol{x})$ is valid when average of difference in the virtual sensing-success ratio between this approximated placement and power-law placement is less than the predefined threshold $\Theta = 0.1$ for both tolerances (i.e., tolerance against random failure and that against battery exhaustion).

4.4. Simple diffusion with concentric centers

Let M_r be the number of concentric circles and r_j be the radius of these circles. Then, the coordinates of centers are $(r_j \cos\theta_k,\ r_j \sin\theta_k)$, $\theta_k = \frac{2\pi k}{M_\theta}$, $(k = 1, \cdots, M_\theta)$. In addition, a center is placed in the same position as the base station. The standard deviation and weights for simple diffusion in the same circle is the same. In this implementation, Eq. (4) is rewritten as follows:

$$F(\boldsymbol{x}) = \sum_{j=1}^{M_r} \frac{p_j}{2\pi\sigma_j^2} \sum_{k=1}^{M_\theta} h(\boldsymbol{x}; \boldsymbol{c}_{jk}, \sigma_j) + \frac{p_{M_r+1}}{2\pi\sigma_{M_r+1}^2} h(\boldsymbol{x}; \boldsymbol{0}, \sigma_{M_r+1}), \tag{6a}$$

$$\boldsymbol{c}_{jk} = (r_j \cos\theta_k,\ r_j \sin\theta_k),\ \theta_k = 2\pi k / M_\theta. \tag{6b}$$

We determined the parameters in Eq. (6) through following strategies.

Since a smaller M_r is better in terms of implementation simplicity, we minimize the value of M_r. We fix the value of M_θ, since M_θ has less impact on the implementation simplicity.

Under the given value of M_r, we formulate the optimization problem to determine the parameters. The objective function and the constraint $\sum_{j=1}^{M_r+1} p_j < P$, $(P \geq 1)$ are same as those in simple diffusion with fixed centers. In addition, we add the constraint $\sigma_j < S$ to consider the upper bound for distance that sensor nodes can reach using simple diffusion. Let R_{max} be the upper bound of distance where sensor nodes can reach using simple diffusion and R be the radius of region D. To place more than 99% of sensor nodes within a circle whose radius is R_{max}, $\sigma_j < \frac{R_{max}}{3R} = S$. This means a smaller S is needed to place sensor nodes in a larger area. The validity of $F(\boldsymbol{x})$ is evaluated in the same way as simple diffusion with fixed centers.

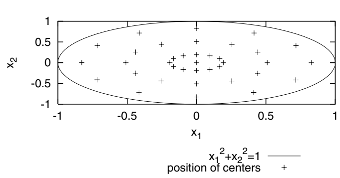

Figure 4. An example of center's positions

5. Performance of each implementation

In this section, we discuss the accuracy and fault tolerance of each implementation. In evaluation, we used a performance metric and simulation scenarios in Sect. 2.2.

5.1. Simple diffusion with fixed centers

We set $P \in \{1.06, 1.10, 1.16\}$. The minimum value of M was 3 for each P. The p.d.f. plots at $x_2 = 0$ are in Fig. 5. In this figure, the density near the border of a unit circle is less than that of power-law placement,since the density of simple diffusion decays very fast at the tail part of the distribution. The larger value of P mitigates the effect of this decay, since a larger P allows more sensor nodes are outside region D. These results indicate that there is a tradeoff between the number of sensor nodes and the accuracy of implementation. Next, let us look at tolerance against random failure. The virtual sensing-success ratio is plotted in Fig. 6. When there are fewer broken nodes, the virtual sensing-success ratio in simple diffusion with fixed centers is lower than that in power-law placement. This difference is larger when P is smaller, since the probability of there being some sensor nodes within the sensing range of the target decreases as P becomes smaller. This is because sensor nodes are less dense near the border of region D as P becomes smaller.

Last, let us look at the tolerance against battery exhaustion. The virtual sensing-success ratio is plotted in Fig. 7. With fewer targets, virtual sensing-success ratio of simple diffusion with fixed centers is lower than that of power-law placement.This is because of the lower density of sensor nodes near the border of region D. The parameter P, on the other hand, has little effect on the period when the virtual sensing-success ratio is reasonable. This is because only the density near the base station affects the existence of alternative routes to it and the difference in density near the base station is not so great between each P value.

We conclude from these simulation results that fault tolerance in simple diffusion with fixed centers is almost the same as that of power-law placement with $\alpha = 0$ when the value of P is appropriately set.

5.2. Simple diffusion with concentric centers

We set $S \in \{0.10, 0.14, 0.18\}$ and $P = 1.1$. The minimum value of M_r was 3 for each S. The p.d.f. plots at $x_2 = 0$ for each S are shown in Fig. 8. In this figure, fluctuations in p.d.f. become higher as the distance from the origin increases. Since the distance between adjacent centers on a circle becomes larger with the radius of a circle, the effect of the tail in simple diffusion increases with the distance from the origin. This characteristic is more remarkable when S is smaller, since the spread of simple diffusion is restricted by the value of S.

Next, let us now look at tolerance against random failure. The virtual sensing-success ratio is plotted in Fig. 9. When S is small, the virtual sensing-success ratio in simple diffusion with concentric centers is lower than that in power-law placement. However, fluctuations in p.d.f. do not affect the virtual sensing-success ratio so much.

Last, let us look at the tolerance against battery exhaustion. The virtual sensing-success ratio is plotted in Fig. 10. The difference between simple diffusion with concentric centers and power-law placement increases with the number of targets. In addition, this difference becomes larger as S becomes smaller. These results indicate that the fluctuations in p.d.f. do not decrease the probability of there being some sensors within the sensing range of the target but do decrease the probability of there being a route to the base station.

We conclude from these results that simple diffusion with concentric centers has almost as high a fault tolerance as power-law placement with $\alpha = 0$ when the value of S is appropriately set.

6. Conclusion

We proposed two ways of achieving power-law placement with $\alpha = 0$ using superposition of simple diffusion. The first way was through simple diffusion with fixed centers that was suitable when the area the sensors were placed was relatively small. The second was through simple diffusion with concentric centers, which could be used even when the area the sensors were placed was large. Both ways demonstrated power-law placement with $\alpha = 0$ with reasonable complexity and accuracy. Through simulation, we also found that the fault tolerance of each implementation was as high as that of power-law placement with $\alpha = 0$ when the control parameters were appropriately set.

This research was partially supported by the Grant-in-Aid for Scientific Research from Ministry of Education, Science, Sports and Culture of Japan, under Grant No. 16310116 (2004-2006).

We would like to thank Ms. Kyoko Ashitagawa for her help with the simulations.

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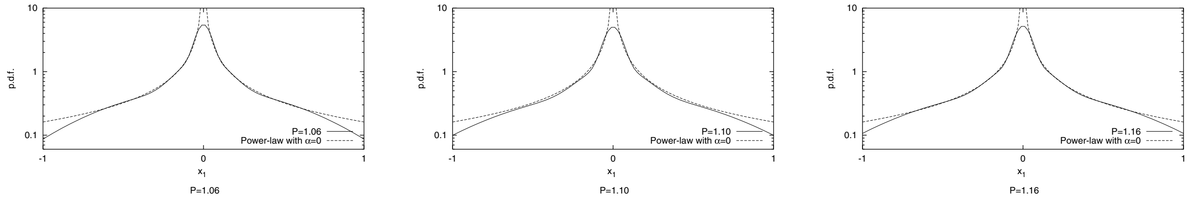

Figure 5. P.d.f. plots of simple diffusion with fixed centers

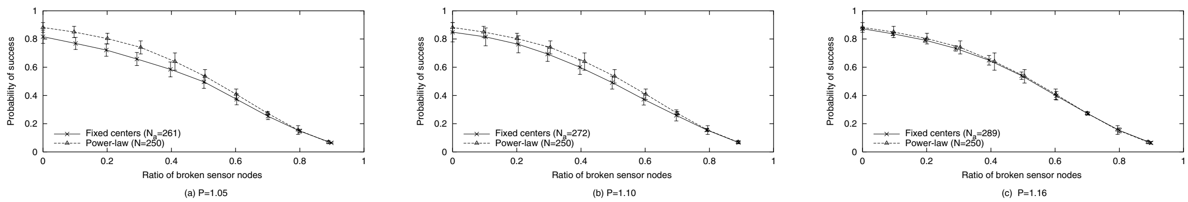

Figure 6. Virtual sensing-success ratio of simple diffusion with fixed centers (random failure)

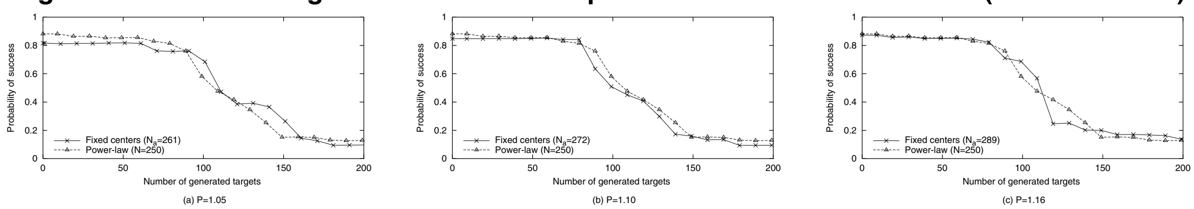

Figure 7. Virtual sensing-success ratio of simple diffusion with fixed centers (battery exhaustion)

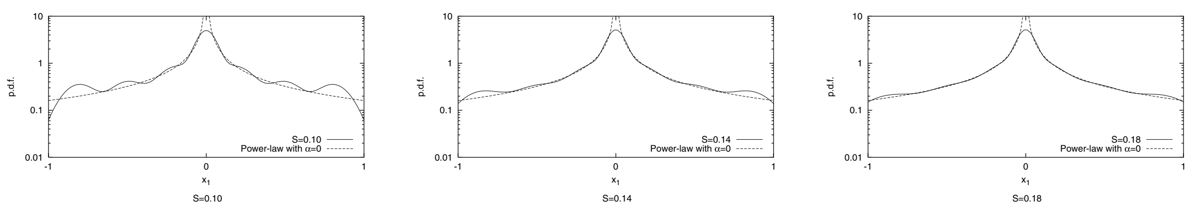

Figure 8. P.d.f. plots of simple diffusion with concentric centers

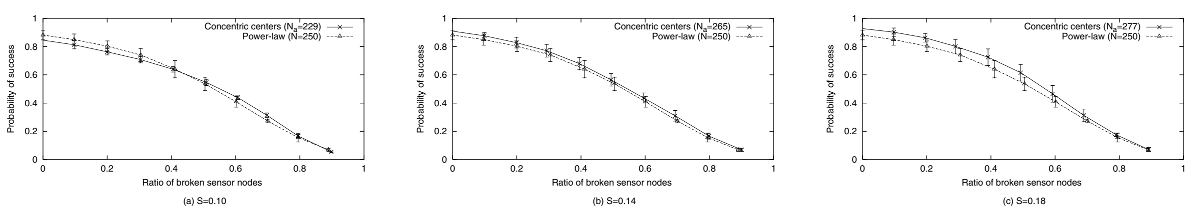

Figure 9. Virtual sensing-success ratio of simple diffusion with concentric centers (random failure)

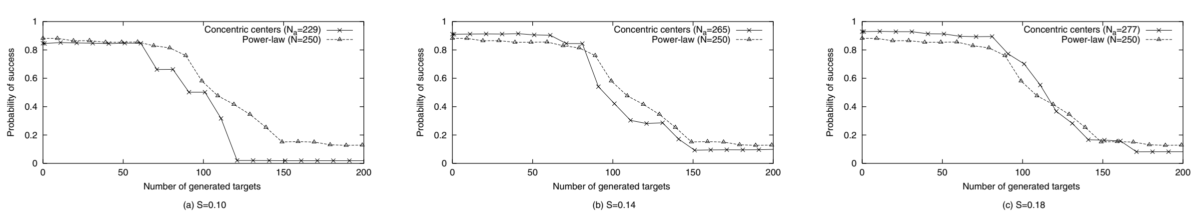

Figure 10. Virtual sensing-success ratio simple diffusion with concentric centers (battery exhaustion)