Estimating Packet Loss-Rate by Using Delay Information and Combinated with Change-of-Meaure Framework

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Abstract—We previously proposed a change-of-measure based performance measurement method which combines active and passive measurement to estimate user-experienced performance. We also applied this method to packet-delay estimation. In this paper, we apply this method to loss-rate estimation. Because packet loss rarely occurs in current networks, its measurement usually requires a huge number of probe packets, which imposes a non-negligible load on the networks. We propose a loss-rate estimation method which requires significantly fewer number of probe packets. In our proposed method, the correlation between delay and loss is measured in advance, and at the time of measurement, the time-averaged loss rate is estimated by using the delay of probe packets and the correlation. We also applied our change-of-measure framework to estimating the loss rate in user packets by using this time-averaged loss rate. We prove that the mean square error in our method is lower than that simple loss measurement, which is estimated by dividing the number of lost packets by the total number of sent packets. We evaluated our method through simulations and actual measurements and found that it can estimate below 10^{-3} packet loss rate with only 900 probe packets.

I. INTRODUCTION

We previously proposed a lightweight and scalable method of measuring performance, *CoMPACT Monitor*, that combines active and passive measurement to estimate performance experienced by users. We also used it to estimate the delay in user packets [1], [2]. In this paper, we report the use of it to estimate the end-to-end loss rate for user packets.

Packet losses significantly degrade the QoS of UDP applications such as streaming video or voice over IP [3],[4]. TCP throughput also depends on a packet loss rate in a large bandwidth-delay product environment [5]. Therefore, it is necessary to measure the packet loss rate to manage the service level of these applications. In terms of managing service level management, we need to know not only the stationary packet loss rate as an indicator of network performance, but also the time-varying loss rates for relatively short time periods (*e.g.*, duration of streaming videos) as service-level statistics. However, there are two drawbacks with current active loss measurement.

First, when loss rate is low, active methods send a huge number of probe packets to detect packet losses that rarely occur. This imposes a non-negligible load on the network, especially when we measure the loss rate for a short period of time. While there are many tools for actively measuring packet loss rate, they only make use of information on lost probe packets and do not use other available packet information. With those methods, the required number of probe packets to measure the loss rate is roughly an order more than the reciprocal of the rate when losses occur independently. When there is a correlation between loss events, which has been reported in many Internet loss measurements [6]-[9], we do not need as many probe packets. Yet, the required number of probe packets is still large and those probe packets may perturb the networks [10].

Second, the loss rate measured by probe packets may not be the same as that experienced by user packets [10]. If we assume that active monitoring measures the time average of network performance and that user traffic is Poissonian, then the performance experienced by users and actively measured performance will be the same. This is a well-known property called PASTA (which stands for "Poisson Arrivals See Time Average") [11]. It is known, however, that current Internet traffic exhibits burstiness and is not Poissonian, in general [12]. In that case, more user packets are transmitted during congested periods, which means that more user packets experience a high loss rate. Thus, the loss rate experienced by users may actually be higher than that measured by those active monitoring. We previously proposed a method that can estimate the performance experienced by users by combining active and passive monitoring [1],[2]. We also applied this method to packet-delay estimation. However, if we simply apply it to estimate loss rate without considering the first drawback, the estimated loss rate may deviate from the actual loss rate when the number of probe packets is limited.

In this paper, we propose a method of estimating packet loss-rate using delay information in probe packets to overcome the first drawback. We use an intuitive expectation that when the delay of a probe packet is large, then the loss probability of user packets sent near the probe packet is high. We propose an estimator for the time-average loss rate that uses the correlation between loss and delay. Then, combining the concept of the *CoMPACT monitor* and this estimator, we propose an estimator for the loss rate experienced by users packets. We prove that the mean square error of our method is smaller than

those of a simple loss measurement. The method was also evaluated through simulations and actual measurements and those showed that it can estimate below 10^{-3} packet loss rate with only 900 probe packets.

The rest of this paper is organized as follows. Section II presents the proposed loss measurement method. Simulations and actual measurement experiments are described in section III. Finally, we give summary in section IV.

II. PROPOSED MEASUREMENT METHOD

Our method of estimating loss rate in user packets involves the following two steps:

- **Step 1:** Estimating the time-average loss rate by using actively measured delay and loss.
- **Step 2:** Converting the time-average loss rate to the loss rate experienced by user packets by using passively measured traffic intensity.

These steps are explained more fully in the next two subsections.

A. Time average loss rate

The main difficulty in this step is measuring rare events with fewer samples (probe packets). We overcome this difficulty by using delay information about probe packets that are not lost.

The objective of this step, time-average loss rate during $(t_1, t_2] LR(t_1, t_2)$, is defined as

$$LR(t_1, t_2) := \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \mathbf{1}_{\{V(t) = D_l\}} dt,$$
(1)

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function, V(t) is the virtual delay of the packet sent at t, D_l is a value set larger than the maximum delay, and $V(t) = D_l$ indicates that the packet is lost. Here, we use the word "virtual" because there may not be any actual user packets at time t [11].

A simple estimator of $LR(t_1, t_2)$ with *n* probe packets sent during $(t_1, t_2]$, SELR(n), is

$$SELR(n) := \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}_{\{Y_i = D_i\}},$$
(2)

where Y_i is the delay for *i*-th probe packet and $Y_i = D_i$ indicates the packet is lost. We also define t_i as the transmission time for the *i*-th probe packet, so $V(t_i) = Y_i$. Actually, if probe packets are sent independently of V(t), *i.e.* in a Poisson manner, (2) is a consistent estimator of (1). However, when seeing an individual estimation for a measurement of a short time period, it may be significantly different from $LR(t_1, t_2)$ when the loss rate is low and the number of probe packets is limited. Now, we can intuitively expect that even if a probe packet is not lost, when the delay for the probe is large, then the loss rate around the time of the probe packet is high as reported in [8]. Therefore, we propose a method of estimation that uses the correlation between the delay in the packet and the loss of neighboring packets.

First, let us define the conditional loss probability $l_c(t, \tau, x)$ as follows:

$$l_c(t, \tau, x) := \Pr[V(t+\tau) = D_l | V(t) = x].$$
 (3)

For large delay x, the conditional loss probability of a packet sent near t is expected to be high and to decrease as $|\tau|$ increases (Fig. 1). Here, we assume that $l_c(t, \tau, x)$ is stationary



(independent of *t*), and denote this as $l_c(\tau, x)$. Then, given the delay of the *i*-th probe packet, Y_i , we obtain the unconditional loss probability for time $t_i + \tau$ as $l_c(\tau, Y_i)$. Therefore, the loss rate in the neighborhood of t_i , $(t_i - \delta_-, t_i + \delta_+]$ $(\delta_-, \delta_+ \ge 0)$, is estimated by

$$\frac{1}{\delta_{+}+\delta_{-}}\int_{-\delta_{-}}^{\delta_{+}}l_{c}(\tau,Y_{i})d\tau.$$
(4)

While $\mathbf{1}_{\{Y_i=D_j\}}$ in (2) only takes 0 or 1, (4) takes value from 0 to 1 even if $Y_i = D_i$. The expectation of (4) in terms of Y_i agrees with $E[LR(t_i - \delta_-, t_i + \delta_+)]$ because

$$E\left[\int_{-\delta_{-}}^{\delta_{+}} l_{c}(\tau, Y_{i}) d\tau\right] = \int_{-\delta_{-}}^{\delta_{+}} \int_{0}^{D_{l}} l_{c}(\tau, x) dF_{i}(x) d\tau$$
$$= \int_{-\delta_{-}}^{\delta_{+}} \Pr[V(t_{i} + \tau) = D_{l}] d\tau$$
$$= E\left[\int_{t_{i} - \delta_{-}}^{t_{i} + \delta_{+}} \mathbf{1}_{\{V(\tau) = D_{l}\}} d\tau\right], \quad (5)$$

where $F_i(x) = \Pr[V(t_i) \le x]$. Therefore, using conditional probability $l_c(\tau, x)$ and *n* measurements of probe packets $\{Y_1, Y_2, \ldots, Y_n\}$ sent in $(t_1, t_2]$, we obtain another estimator for $LR(t_1, t_2)$ by changing $\mathbf{1}_{\{Y_i = D_i\}}$ in *SELR* to (4) as

$$ELR(n, \delta_{+}, \delta_{-}) := \frac{1}{n} \sum_{i=1}^{n} \frac{1}{\delta_{+} + \delta_{-}} \int_{-\delta_{-}}^{\delta_{+}} l_{c}(\tau, Y_{i}) d\tau.$$
(6)

Compared to (2) which only uses the loss rate in timing the probe packet transmission, the estimator (6) is expected to be more accurate because it uses the loss rate of the neighborhood of the probe packets by using the delay in probe packets and the correlation between the delay and loss probability. Actually, we can prove that the mean square error in the proposed method is smaller than that in simple loss rate estimation (2). Here, we compare the error of the estimation when using one probe packet; $E[(ELR(1, \delta_+, \delta_-) - LR(-\delta_-, \delta_+))^2]$. Errors for multiple packets can roughly be obtained by dividing the MSE by the number of probe packets if we can assume that error between probe packets is independent. Let R(x) be $R(x) := \int_{-\delta}^{\delta_+} l_c(t,x) dt/(\delta_+ + \delta_-)$. Then,

$$E[(ELR(1, \delta_{+}, \delta_{-}) - LR(-\delta_{-}, \delta_{+}))^{2}]$$

$$= \int_{0}^{D_{l}} E\left[\left(R(x) - \frac{\int_{-\delta_{-}}^{\delta_{+}} \mathbf{1}_{\{V(t) = D_{l} | V(0) = x\}} dt}{\delta_{+} + \delta_{-}}\right)^{2}\right] dF_{1}(x)$$

$$= \int_{0}^{D_{l}} Var\left[R(x) - \frac{\int_{-\delta_{-}}^{\delta_{+}} \mathbf{1}_{\{V(t) = D_{l} | V(0) = x\}} dt}{\delta_{+} + \delta_{-}}\right] dF_{1}(x)$$

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$$+ \int_{0}^{D_{l}} \mathbb{E} \left[R(x) - \frac{\int_{-\delta_{-}}^{\delta_{+}} \mathbf{1}_{\{V(t) = D_{l} | V(0) = x\}} dt}{\delta_{+} + \delta_{-}} \right]^{2} dF_{1}(x)$$

$$= \int_{0}^{D_{l}} \operatorname{Var} \left[\frac{\int_{-\delta_{-}}^{\delta_{+}} \mathbf{1}_{\{V(t) = D_{l} | V(0) = x\}} dt}{\delta_{+} + \delta_{-}} \right] dF_{1}(x).$$
(7)

On the other hand, the mean square error for the *SELR* is obtained as follows. Let $f_1(D_l)$ as the stationary loss probability as $f_1(D_l) := \mathbb{E}[\mathbf{1}_{\{V(0)=D_l\}}] = 1 - F_1(D_l)$. Then, $\mathbb{E}[(SELP(1) - LP(-\delta - \delta))^2]$

$$E[(SELR(1) - LR(-\delta_{-}, \delta_{+}))^{2}]$$

$$= \int_{0}^{D_{l}} \operatorname{Var} \left[\frac{\int_{-\delta_{-}}^{\delta_{+}} \mathbf{1}_{\{V(t) = D_{l} | V(0) = x\}} dt}{\delta_{+} + \delta_{-}} \right] dF_{1}(x)$$

$$+ f_{1}(D_{l}) + \int_{0}^{D_{l}-} E \left[\frac{\int_{-\delta_{-}}^{\delta_{+}} \mathbf{1}_{\{V(t) = D_{l} | V(0) = x\}} dt}{\delta_{+} + \delta_{-}} \right]^{2} dF_{1}(x)$$

$$- 2f_{1}(D_{l}) E \left[\frac{\int_{-\delta_{-}}^{\delta_{+}} \mathbf{1}_{\{V(t) = D_{l} | V(0) = D_{l}\}} dt}{\delta_{+} + \delta_{-}} \right]$$

$$= \int_{0}^{D_{l}} \operatorname{Var} \left[\frac{\int_{-\delta_{-}}^{\delta_{+}} \mathbf{1}_{\{V(t) = D_{l} | V(0) = x\}} dt}{\delta_{+} + \delta_{-}} \right] dF_{1}(x)$$

$$+ f_{1}(D_{l}) + \int_{0}^{D_{l}-} R(x)^{2} dF_{1}(x) - 2f_{1}(D_{l})R(D_{l}). \quad (8)$$

Therefore, the difference between (8) and (7) is

$$f_1(D_l) \left(1 - R(D_l)\right)^2 + \int_0^{D_l} R(x)^2 dF_1(x) \ge 0, \quad (9)$$

and the error of our estimator is smaller or equal to that of a simple loss estimator. The first term equals zero if the mean packet loss probability near the lost probe packet is one. The second term is zero if the mean loss probability near the probe packet is large. Although packet loss on the Internet exhibits a high degree of burstiness [9], a situation where both terms equal zero can hardly be expected, especially for a relatively large δ .

Our estimator (6) requires conditional loss probability $l_c(\tau, x)$. This can be obtained through either preprocessing or in an online way. In preprocessing, probe packets are sent with short intervals, and we can take the loss and delay pairs of the interval. The interval should be sufficiently short that the integral in (6) can be approximated by the sum of the intervals. Then, $l_c(\tau, x)$ can be calculated using the loss and delay pairs. We expect this probability remains almost unchanged in a network path during a period of time that can be considered as stationary (such as a busy period). Thus, once the $l_c(\tau, x)$ of a period of time has been obtained, we can use this for other measurements done in the same period of time. However, to cope with gradual changes in network conditions, introducing an online update of $l_c(\tau, x)$ may be better. In that case the delay and loss pairs for different intervals can be obtained by sending probe packets with different intervals (e.g. Poisson process), while simultaneously measuring the loss rate. Then,

 $l_c(\tau, x)$ is updated with these delay and loss pairs, for example, by taking a moving average.

B. Estimation of user packet loss rate

This step uses our previously proposed method to estimate the delay in user packets [1], [2]. In this subsection, we present a brief review of it and its combination with the results obtained in Step 1. The mathematical foundation of the method can be found in [1], [2].

Our proposed change-of-measure based network performance measurement method, *CoMPACT Monitor*, was aimed at estimating the network performance experienced by users. To do this, it actively measures the performance through probe packets, and passively measures the number of user packets sent near the probe packets. Then, by changing the measure for the performance of probe packets to that of user packets, we obtain an estimator of network performance experienced by users.

The objective of this step is to estimate the loss rate experienced by user packets sent in $(t_1, t_2]$, $ULR(t_1, t_2)$:

$$ULR(t_1, t_2) = \frac{\int_{t_1}^{t_2} \mathbf{1}_{\{V(t) = D_l\}} dA(t)}{\int_{t_2}^{t_2} dA(t)},$$
(10)

where A(t) is the arrival process for user packets. Here, the denominator represents the number of user packets sent in $(t_1, t_2]$ and the numerator represents the number of lost user packets.

Modeling A(t) as a fluid, we demonstrated that an empirical distribution of the delay for any user fluid A(t) could be obtained using actively and passively measured values [1]. Let a(i) be user traffic intensity at t_i , *i.e.*, dA(t)/dt at the *i*-th active measurement timing. The a(i) is obtained through passive measurement by counting the number of user packets. Assuming that measurement timing is stationary, it can be proved for any $D \in \mathbf{R}_+$ that

$$\lim_{t_2 \to \infty} \frac{\int_{t_1}^{t_2} \mathbf{1}_{\{V(t) > D\}} dA(t)}{\int_{t_2}^{t_2} dA(t)} = \lim_{n \to \infty} \frac{\sum_{i=1}^{n} \mathbf{1}_{\{Y_i > D\}} a(i)}{\sum_{i=0}^{n} a(i)} a.s.$$
(11)

Therefore, by setting *D* to D_l , a(i) as the number of user packets sent in $(t_i - \delta_-, t_i + \delta_+]$, and *u* as $\sum_{i=1}^n a(i)$, we obtain an estimator of (10) by simply applying the change-of-measure method as

$$SEULR(n) := \frac{1}{u} \sum_{i=1}^{n} \mathbf{1}_{\{Y_i = D_l\}} a(i).$$
(12)

Compared with (2), the estimator (12) weights the event $Y_i = D_l$ by a(i) to convert the time-average loss rate to user-experienced loss rate. *SELR* has been proved to agree with *ULR* if the measurement lasts long enough. However, because it assumes that user packets sent near the lost probe packet will all be lost, otherwise no user packets are lost, it shares the same problem as (2). Therefore, by combining the results of Step 1 and the change-of-measure based method, we propose an estimator to determine the loss rate for user packets *EULR*(n, δ_+, δ_-) as

packets $EOLK(n, \delta_+, \delta_-) := \frac{1}{u} \sum_{i=1}^n \frac{a(i)}{\delta_+ + \delta_-} \int_{-\delta_-}^{\delta_+} l_c(\tau, Y_i) d\tau.$ (13) This estimator is obtained by changing $\mathbf{1}_{\{Y_i = D_l\}}$ in (12) to $\frac{1}{\delta_+ + \delta_-} \int_{-\delta_-}^{\delta_+} l_c(\tau, Y_i) d\tau$ the same as the relationship between (2) and (6).



A. Simulation results III. EVALUATION

We evaluated our method through simulations for the network in Fig. 2. User sources generate exponential On-Off traffic, whose means are 1 s and 14 s, respectively. During the On period, 1,000-byte packets were sent with an exponentially distributed interval with a mean of 10.6 ms. Thus, each source sent packets at 50 kbps on average. The number of sources was 140, so that the utilization rate of the intermediate link was 0.7. The buffer capacity of the router was set to 50 packets.

To obtain conditional loss probability, $l_c(\tau, x)$, we first ran a 50,000-s simulation in advance where 64-byte probe packets were sent with an exponential interval with a mean of 10 ms. The $l_c(\tau, x)$ were calculated using the loss and delay information of these probe packets. After calculating $l_c(\tau, x)$, we ran 10 simulations each lasting 1,000 s and calculated ELR and EULR. In each run, probe and user packets were sent in the same way as in the first 50,000-s simulation. Time-average loss rate, LR, was calculated using all probe packets, and useraverage loss rate, ULR, was calculated using all 1,000-byte packets. To estimate loss rate, we only used one probe packet per 100 probe packets. In other words, we only sent one probe packet every second on average during actual measurements. We set $(\delta_+, \delta_-) = (100, 100)$ (ms) in this paper. The maximum delay was 29 ms. Figure 1 shows $l_c(\tau, 0)$ and $l_c(\tau, 20)$ when the number of hosts is 140. As expected, $l_c(\tau, 20)$ decreased as τ increased. We can also see that $l_c(\tau,0)$ increased as τ increased and converged to a constant, which is expected to be the time-average loss rate (unconditional loss probability).

First, we tested ELR, the estimator for time-average loss rate. Figure 4 shows LR (time-average loss rate), SELR (estimates using only the loss information of probe packets), and ELR (estimates using both delay and loss information of probe packets) for ten simulations. We see that while SELRs deviated from LRs, ELR could estimate LR with high accuracy. Note



Fig. 4. Loss rate and its estimation for ten simulations.



Simulation run Loss rate for user packets and its estimations results for ten Fig. 6. simulations

that while we used the same $l_c(\tau, x)$ for all estimates, *ELR* could estimate different values for LR for ten simulations, by reflecting difference in probe delays. We can also see that no probe packets were lost in simulations 2, 3, 5 and 6, while ELR could estimate loss rate using by delay information. We also compared mean square errors, $E[(ELR - LR)^2]$ and $E[(SELR - LR)^2]$ by varying t_2 from 200 to 1,000 s (we fixed t_1 at 0). Figure 5 shows the results. The accuracy of our method was higher than that for the simple loss rate estimation as proved in (9).

Next, we tested EULR, the estimator of the loss rate in user packets. Figure 6 shows the ULR (user-experienced loss rate), SEULR (estimates of ULR using only the loss information of probe packets and number of user packets), and EULR (estimates of ULR using both delay and loss information of probe packets and number of user packets). We can also see from this figure that our estimation could follow the loss rate for user packets, which was about 1%, i.e. higher than the time-average loss rate shown in Fig. 4. This was due to the correlation between the number of user packets and the loss rate, which was discussed in Section I. By weighting the loss rate with the number of user packets sent near probe packets, our method could convert the time-average loss rate to an useraverage loss-rate. Figure 7 shows the mean square error of the EULERs and SEULRs to ULRs. Our estimator achieved a lower mean square error compared with the simple changeof-measure method.

B. Actual measurement results

We did end-to-end loss and delay measurements on an actual network to evaluate our method. We measured oneway delay and loss from an asymmetric digital subscriber line (ADSL) customer to a company LAN during office hours (10:00-18:00) in February 2003. The path consisted of 15 hops



Fig. 7. Mean square errors for user-experienced loss rate.



Fig. 8. Sample path of delay and loss in measurement (Losses are shown as 250 ms delay)



Fig. 9. Conditional loss probability $(l_c(\tau, x))$ for x = 70 and 170 (ms).

and the narrowest link along the path was the ADSL up-link, with a bandwidth of about 400 Kbps¹. Here, we only evaluated the estimator for time-average loss rate because there were no user packets on the link.

We sent 64-byte UDP packets between two GPSsynchronized PCs in both LANs as a Poisson process where the mean interval was 20 ms. We ran thirty measurements each lasting 900 s and used the first twenty to calculate conditional loss probability, and the last ten to estimate loss rate. The maximum, mean and minimum delay were 207, 60, and 20 ms, respectively. The time-average loss rate for the whole measurement was 0.07 %.

Figure 8 shows a sample path for the delay and loss of the probe packets (Here, loss is shown as 250 ms delay). We can see the fluctuations in delay, and packet losses during some peaks of fluctuation. This indicates that there is a correlation between the loss and delay even in the actual network. To provide more direct evidence of this correlation, Fig. 9 has conditional loss probability for delays of 70 ms and 170 ms. There is a clear correlation between loss and delay the same as in simulation.

Figure 10 shows *LR*, *SELR*, and *ELR* for 10 measurements where *LR* was calculated using all probe packets, and *ELR* and *SELR* was estimated by using one probe packets per 50 packets. In these simulations, as the loss rate varied from 10^{-5} to 10^{-3} , we plotted the loss rate semi-logarithmically. Except for measurements 2 and 3, no probe packets were lost, and the *SELR* was zero, which cannot be shown in the figure. Even so, our method could estimate time-average loss rate accurately especially for loss rates over 10^{-4} . Figure 11 shows the mean square errors of proposed estimator *ELR* and simple loss estimator *SELR*. The errors for *ELR* are smaller than those for *SELR* for every number of probe packets.

IV. CONCLUSION

We proposed a two-step method of estimating the loss rate in user packets that involved: 1) estimating the time-



average loss rate and 2) converting that rate to the userexperienced loss rate. We used the delay in probe packets and the conditional loss probability given by their delay in Step 1. In Step 2, the time-average loss rate estimated in Step 1 was converted to the user-experienced loss rate through our previously proposed change-of-measure based method. Our method can be used to estimate both the time-average and user-experienced loss rate accurately with a limited number of probe packets. It can be used to determine the average loss rate over short periods of time such as duration of a streaming videos or phone calls.

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¹During measurements, because no packets except active probe packets were sent to the link, it could not cause queueing delays or losses for probe packets on the link.